PROBLEM SET #1

DUE SEPTEMBER 8

1. Find 2 distinct bases for $\mathbb{Q}^2$.

2. Find all 3 bases of $\mathbb{F}_2^2$.

3. Are the vectors $(1, 1, 1)$, $(0, -1, 2)$, $(2, 1, 3)$, and $(4, 0, 11)$ linearly independent? If not, find a dependence relation.

4. Let $a, b \in \mathbb{F}$ and let $\vec{v}, \vec{w} \in V$. Show that

$$(a + b)(\vec{v} + \vec{w}) = a\vec{v} + a\vec{w} + b\vec{v} + b\vec{w}.$$

5. Let $C(\mathbb{R})$ be the collection of all continuous functions from $\mathbb{R}$ to itself. Let C_+ denote the subset of "even" functions:

$$C_+ = \{f | f(-x) = f(x)\},$$

and let C_- denote the "odd" functions:

$$C_- = \{f | f(-x) = -f(x)\}.$$

Show that both are subspaces of $C(\mathbb{R})$. Show also that $C(\mathbb{R}) = C_+ \oplus C_-$.

6. Let V be the vector space of all infinite sequences of real numbers. Show that the following are subspaces:

(1) ℓ_1, the subset of absolutely convergent sequences:

$$\ell_1 = \{(a_1, \dots) | \sum |a_n| < \infty\}.$$

(2) z_0, the subset of eventually zero sequences.
(3) c_0, the subset of sequences which converge to zero.
(4) c, the subset of sequence which converge.
(5) ℓ_∞, the subset of bounded sequences:

$$\ell_\infty = \{(a_1, \dots) | \sup |a_n| < \infty\}.$$