PROBLEM SET #1: DUE THURSDAY, FEBRUARY 7$^{\text{TH}}$

Book Problems

Chapter 1: 3, 13, 18, 20, 21, 22, 25

Other Problems

(1) Let A, B, and C be sets. Assume we are also given maps $f\colon A \to C$ and $g\colon B \to C$. We define the **fiber product** $A \times_C B$ as the subset of $A \times B$ given by

$$A \times_C B = \{(a, b) | f(a) = g(b)\}.$$

We note that as a subset of the product, we still have natural maps

$$\pi_A\colon A \times_C B \to A, \text{ and } \pi_B\colon A \times_C B \to B.$$

Show that the fiber product has the following universal property: given any D together with maps $h\colon D \to A$ and $k\colon D \to B$ such that $f \circ h = g \circ k$, there is a unique map $H\colon D \to A \times_C B$ such that $h = \pi_A \circ H$ and $k = \pi_B \circ H$. Better said, there is an H making the following diagram commute:

$$\begin{tikzcd}
D \arrow[dr, dotted] \arrow[drr, bend left, "h"] \arrow[ddr, bend right, "k"'] & & \\
& A \times_C B \arrow[r, "\pi_A"'] \arrow[d, "\pi_B"] & A \arrow[d, "f"] \\
& B \arrow[r, "g"'] & C
\end{tikzcd}$$

In other words, if we filled in the partial square we had in Lecture #1 to a square, then we get the same basic universal property.