

**PROBLEM SET #1: DUE THURSDAY, FEBRUARY 7<sup>TH</sup>**

BOOK PROBLEMS

Chapter 1: 3, 13, 18, 20, 21, 22, 25

OTHER PROBLEMS

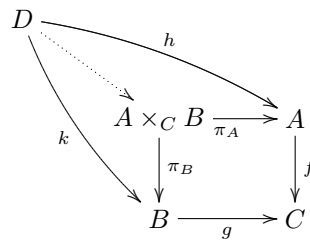
- (1) Let  $A$ ,  $B$ , and  $C$  be sets. Assume we are also given maps  $f: A \rightarrow C$  and  $g: B \rightarrow C$ . We define the **fiber product**  $A \times_C B$  as the subset of  $A \times B$  given by

$$A \times_C B = \{(a, b) | f(a) = g(b)\}.$$

We note that as a subset of the product, we still have natural maps

$$\pi_A: A \times_C B \rightarrow A, \text{ and } \pi_B: A \times_C B \rightarrow B.$$

Show that the fiber product has the following universal property: given any  $D$  together with maps  $h: D \rightarrow A$  and  $k: D \rightarrow B$  such that  $f \circ h = g \circ k$ , there is a unique map  $H: D \rightarrow A \times_C B$  such that  $h = \pi_A \circ H$  and  $k = \pi_B \circ H$ . Better said, there is an  $H$  making the following diagram commute:



In other words, if we filled in the partial square we had in Lecture #1 to a square, then we get the same basic universal property.