DATE: Nov. 14, Wednesday
TIME: 10:00 am - 10:50 am

Please write clearly, reduce answers to their simplest form, and box your answers.
To receive full credit you must show ALL your work.

Student's Name (Please print): __________________________________________

Pledge: On my honor as a student at the University of Virginia I have neither given nor received aid on this test.

Signature: _______________________________

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Problem 1 (25 points)
Find the volume of the region $E$ bounded above by $x^2 + y^2 + z^2 = 2$ and below by $z = x^2 + y^2$. 
Problem 2 (25 points)
Consider the region $R$ in $\mathbb{R}^2$ bounded by $x^2 - 2xy + 5y^2 = 1$, and the transformation $T$ given by 
\[ x = u + \frac{v}{2}, \quad y = \frac{v}{2}. \]

(a) (10 points) Find, describe and sketch the region $S$ in the $uv$-plane corresponding to $R$ (via the transformation $T$ in the sense that $T : S \to R$).

(b) (5 points) Find the Jacobian of $T$ (Use the proper notation!).

(c) (10 points) Evaluate 
\[ I = \iint_R \sqrt{x^2 - 2xy + 5y^2} \, dA \]
using the transformation $T$. 
Problem 3 (25 points)
Find the work done by the force field \( \mathbf{F}(x, y) = 3x^2 \mathbf{i} + (4x + y^2) \mathbf{j} \) on a particle that moves along the following paths:

(a) (10 points) \( C_1 \) is the line segment from \((1, 0)\) to \((0, 1)\).

(b) (15 points) \( C_2 \) is part of the curve \( x^2 + y^2 = 1 \) for which \( x \geq 0 \) and \( y \geq 0 \) (the particle moves counterclockwise).
**Problem 4** (15 points)
Find the mass of a ball given by $x^2 + y^2 + z^2 \leq 9$ if the density at any point, denoted by $D(x, y, z)$, is proportional to its distance from the origin.

**Problem 5** (10 points)
Using cylindrical coordinates set up, but do not evaluate the integral

$$I = \iiint_E dV,$$

where $E$ is the region bounded above by $x^2 + y^2 + z^2 = 4$ and below by $z = \sqrt{2}$. 
**Bonus Problem 6** (10 points)
Solve Problem 5 using spherical coordinates.