MATH 231, Calculus III, Section 1, Fall 2007

Test 3

Date: Nov. 14, Wednesday Time: 10:00 am - 10:50 am

Please write clearly, reduce answers to their simplest form, and box your answers. To receive full credit you must show ALL your work.

Student's Name (Please print):

Pledge: On my honor as a student at the University of Virginia I have neither given nor received aid on this test.

Signature: _____

Problem	Points	Score
1	25	
2	25	
3	25	
4	15	
5	10	
6 (Bonus)	10	
Total	110/100	

Problem 1 (25 points) Find the volume of the region E bounded above by $x^2 + y^2 + z^2 = 2$ and below by $z = x^2 + y^2$. **Problem 2** (25 points) Consider the region R in \mathbb{R}^2 bounded by $x^2 - 2xy + 5y^2 = 1$, and the transformation T given by $x = u + \frac{v}{2}, y = \frac{v}{2}$.

(a) (10 points) Find, describe and sketch the region S in the *uv*-plane corresponding to R (via the transformation T in the sense that $T: S \to R$).

(b) (5 points) Find the Jacobian of T (Use the proper notation!).

(c) (10 points) Evaluate

$$I = \iint\limits_R \sqrt{x^2 - 2xy + 5y^2} \, dA$$

using the transformation T.

Problem 3 (25 points)

Find the work done by the force field $\mathbf{F}(x, y) = 3x^2\mathbf{i} + (4x + y^2)\mathbf{j}$ on a particle that moves along the following paths:

(a) (10 points) C_1 is the line segment from (1,0) to (0,1).

(b) (15 points) C_2 is part of the curve $x^2 + y^2 = 1$ for which $x \ge 0$ and $y \ge 0$ (the particle moves counterclockwise).

Problem 4 (15 points)

Find the mass of a ball given by $x^2 + y^2 + z^2 \le 9$ if the density at any point, denoted by D(x, y, z), is proportional to its distance from the origin.

Problem 5 (10 points) Using cylindrical coordinates set up, but do not evaluate the integral

$$I = \iiint_E dV,$$

where E is the region bounded above by $x^2 + y^2 + z^2 = 4$ and below by $z = \sqrt{2}$.

Bonus Problem 6 (10 points) Solve Problem 5 using spherical coordinates.