# MATH 231, Calculus III, Section 1, Fall 2007

## Test 2

Date: Oct. 24, Wednesday Time: 10:00 am - 10:50 am

#### Please write clearly, reduce answers to their simplest form, and box your answers. To receive full credit you must show ALL your work.

Student's Name (Please print):

Pledge: On my honor as a student at the University of Virginia I have neither given nor received aid on this test.

Signature: \_\_\_\_\_

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
5 (Bonus)	10	
Total	110/100	

#### Problem 1 (25 points)

(a) (8 points) Evaluate the double integral

$$\iint_{R} (4-x)dA, \quad \text{where} \quad R = \{(x,y) \in \mathbb{R}^{2} | \ 0 \le x \le 4, 0 \le y \le 3\},\$$

by first identifying it as the volume of a solid.

(b) (8 points) If z = z(x, y) is given by  $\sin(xyz) = x + 2y + 3z$ , find  $\frac{\partial z}{\partial x}$ .

(c) (9 points) Let  $z = z(u^2 - v^2, v^2 - u^2)$ . Show that

$$u\frac{\partial z}{\partial v} + v\frac{\partial z}{\partial u} = 0.$$

### Problem 2 (25 points)

Find the directions in which the directional derivative of the function  $f(x, y) = x^2 + \sin(xy)$  at the point (1, 0) has the value 1.

**Problem 3** (25 points) Evaluate the following double integral:

$$I = \iint_D \sin(xy) \, dA,$$

where D is bounded by y = 1, y = 2, the y-axis and  $x = \frac{\pi}{y}$ .

**Problem 4** (25 points) Evaluate the following double integral:

$$I = \iint_{D} \frac{y \, dA}{\sqrt{x^2 + y^2 - (x^2 + y^2)^2}},$$

where  $D = \{(x, y) \in \mathbb{R}^2 | x \ge 0, y \ge 0, \frac{1}{2} \le x^2 + y^2 \le 1\}.$ 

#### Bonus Problem 5 (10 points)

Suppose that f is a differentiable function of three variables. Show that the maximum value of the directional derivative of f in the direction of  $\mathbf{u}$  is  $|\nabla f(x, y, z)|$  and it occurs when  $\mathbf{u}$  has the same direction as the gradient vector of f.