Date: Sep. 26, Wednesday  
Time: 10:00 am - 10:50 am

Please write clearly, reduce answers to their simplest form, and box your answers.  
To receive full credit you must show ALL your work.

Student's Name (Please print): __________________________________________

Pledge: On my honor as a student at the University of Virginia I have neither given nor received aid on this test.

Signature: _______________________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>8 (Bonus)</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>110/100</strong></td>
<td></td>
</tr>
</tbody>
</table>
**Problem 1** (10 points)
Let the vectors \( \mathbf{a} \) and \( \mathbf{b} \) be given by \( \mathbf{a} = j - k \) and \( \mathbf{b} = 2i + j + 3k \). Find:

(a) (2 points) the dot product of \( \mathbf{a} \) and \( \mathbf{b} \)

(b) (4 points) the cross product of \( \mathbf{a} \) and \( \mathbf{b} \)

(c) (4 points) the angle between the vectors \( \mathbf{a} \) and \( \mathbf{b} \)

---

**Problem 2** (15 points)
The velocity of a particle is given by \( \mathbf{v}(t) = \cos(2t)i + \sin(t)j \). If the position of the object at time \( t = \frac{\pi}{4} \) is at the point \((2, 0)\), find its position vector.
Problem 3 (15 points)
(a) (8 points) Find parametric equations of the line $L_1$ through the point $(-1, 0, 2)$ and parallel to the line $L_2$ given by the equations $x + 2 = \frac{1}{3}y = z + 1$.

(b) (7 points) Find an equation of the plane $P_1$ through the point $(6, 4, -3)$ and parallel to the plane $P_2$ given by $3x - y + 2z - 5 = 0$.

Problem 4 (10 points)
Find the point where the curves $r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle$ and $r_2(s) = \langle 3 - s, s - 2, s^2 \rangle$ intersect each other.
Problem 5 (15 points)
Find and describe the domain of the function

\[ f(x, y, z) = \ln(16 - 4x^2 - 4y^2 - z^2). \]

Then sketch the domain, labeling its intersections with the coordinate axes.

Problem 6 (10 points)
Let

\[ f(x, y) = \frac{x^2 + xy + y^2}{x^2 + y^2}. \]

Find the limit \( L = \lim_{(x,y) \to (0,0)} f(x, y) \), if it exists, or show that the limit does not exist.
Problem 7 (25 points)
Given the vector \( \mathbf{r}(t) = \langle \frac{\sqrt{2}}{2} t^2, -\frac{1}{3} t^3, t \rangle \), find

(a) (10 points) the arc length function from \( t = 1 \)

(b) (5 points) the unit tangent vector

(c) (10 points) the unit normal vector at \( t = 0 \)
**Bonus Problem 8** (10 points)
Show that if $|\mathbf{r}(t)| = c$ (a constant), then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all $t$. 