MATH 231, Calculus III, Section 1, Fall 2007

Test 1

Date: Sep. 26, Wednesday
Time: 10:00 am - 10:50 am

Please write clearly, reduce answers to their simplest form, and box your answers.
To receive full credit you must show ALL your work.

Student's Name (Please print): ____

Pledge: On my honor as a student at the University of Virginia I have neither given nor received aid on this test.

Signature: __________________________________

Problem	**Points**	**Score**
1	10	
2	15	
3	15	
4	10	
5	15	
6	10	
7	25	
8 (Bonus)	10	
Total	**110/100**	

Problem 1 (10 points)
Let the vectors $\mathbf{a}$ and $\mathbf{b}$ be given by $\mathbf{a} = \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$. Find:

(a) (2 points) the dot product of $\mathbf{a}$ and $\mathbf{b}$

(b) (4 points) the cross product of $\mathbf{a}$ and $\mathbf{b}$

(c) (4 points) the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$

Problem 2 (15 points)
The velocity of a particle is given by $\mathbf{v}(t) = \cos 2t\mathbf{i} + \sin t\mathbf{j}$. If the position of the object at time $t = \frac{\pi}{4}$ is at the point $(2, 0)$, find its position vector.

Problem 3 (15 points)
(a) (8 points) Find parametric equations of the line L_1 through the point $(-1, 0, 2)$ and parallel to the line L_2 given by the equations $x + 2 = \frac{1}{3}y = z + 1$.

(b) (7 points) Find an equation of the plane P_1 through the point $(6, 4, -3)$ and parallel to the plane P_2 given by $3x - y + 2z - 5 = 0$.

Problem 4 (10 points)
Find the point where the curves $\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle$ and $\mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$ intersect each other.

Problem 5 (15 points)
Find and describe the domain of the function

$$f(x, y, z) = \ln(16 - 4x^2 - 4y^2 - z^2).$$

Then sketch the domain, labeling its intersections with the coordinate axes.

Problem 6 (10 points)
Let

$$f(x, y) = \frac{x^2 + xy + y^2}{x^2 + y^2}.$$

Find the limit $L = \lim_{(x,y)\to(0,0)} f(x, y)$, if it exists, or show that the limit does not exist.

Problem 7 (25 points)
Given the vector $\mathbf{r}(t) = \langle \frac{\sqrt{2}}{2}t^2, -\frac{1}{3}t^3, t \rangle$, find

(a) (10 points) the arc length function from $t = 1$

(b) (5 points) the unit tangent vector

(c) (10 points) the unit normal vector at $t = 0$

Bonus Problem 8 (10 points)
Show that if $|\mathbf{r}(t)| = c$ (a constant), then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all t.