SHORT REVIEW FOR MATH 231 FINAL

Geometry: dot products, cross products, scalar triple product ; interpretation as area of parallelogram, volume of parallelpiped.
Euclidean/Cartesian, Polar, Cylindrical, Spherical Coordinates and their volume elements $dxdydz, rdrd\theta, \rho^2 \sin(\phi) d\rho d\phi d\theta$.
Parametric equations of curves $C : \mathbf{r}(t)$: (1) line through P in direction v $[P + tv]$, (2) line segment from P to Q $[(1-t)P + tQ]$. Tangent line to curve at P. Level curves $f(x, y) = k$.
Parametric equations of surfaces $S : \mathbf{R}(u, v) = x(u, v), y(u, v), z(u, v))$: (1) plane through P_0 with normal vector $\mathbf{n}$ $[(P - P_0) \bullet \mathbf{n} = 0]$, (2) sphere $\rho = a$ in spherical, (3) cylinder $r = k$ in cylindrical, (4) graph $z = g(x, y)$ or $y = g(x, z)$.
Tangent plane to surface at $P_0 = r(u_0, v_0)$: $\mathbf{n} = \mathbf{r}_u \times \mathbf{r}_v$; graph $z = g(x, y)$ has $\mathbf{n} = (-g_x, -g_y, 1)$. Level surfaces $f(x, y, z) = k$.
Area of surface $A(S) = \int\int_S 1 d\mathbf{S}$; surface integral of function $\int\int_S f(x, y, z) d\mathbf{S}$
$= \int\int_D f(r(u, v)\, r_u \times r_v \, dA$.

Functions: vector-valued $r(t)$, tangent vector, $r'(t)$; scalar-valued functions, partial derivatives, directional derivatives $D_u(f)$, gradient; differentiable functions, directional derivatives given by gradient dot u; Clairaut's Theorem (mixed partials agree if continuous). Chain Rule. Gradient as direction of maximum increase of function, with $\| \bigtriangledown (f)(\vec{x})\|$ the maximum rate of increase at $\vec{x}$; tangent plane to $z = f(x, y)$ or to implicitly to a level surface $f(x, y, z) = k$.
Local extrema occur at critical points or on the boundary [No Lagrange Multipliers this test!!]; a continuous function on a closed bounded set attains its absolute maximum and minimum on that set.

Integration: Integration in Euclidean, polar, cylindrical, spherical coordinates. Change of variables, transformations, Jacobians. Change of iteration. Line integrals, work. Surface integrals.

Regions: R open [all points P in R have ball $B_\varepsilon(P)$ contained in R; usually $<$ inequalities], connected [there is a continuous curve from P to Q for any two points in R], simply connected [connected and any closed path in R can be continuosly shrunk in R to a point, i.e. no holes], closed [all limit points of sequences in R stay in R; usually $\leq$ inequalities].
Examples: open/closed disk , annulus, sphere, ball.

Vector Fields: $\mathbf{F}$, gradients $\mathbf{F} = Grad(f) = \nabla f$, curl $curl(\mathbf{F}) = \nabla \times \mathbf{F}$, divergence $\nabla \bullet \mathbf{F}$, Curl(Grad(f)) = 0, Div(Curl($\mathbf{F}$)) = 0. Conservative ($\mathbf{F}$ = Grad(f)) iff mixed partials agree [when R open, simply connected], iff (closed line integrals zero) iff (line integrals independent of path) [when R open, connected].
Oriented surface [smooth choice of normal vector], surface integral of vector field $\int\int_S \mathbf{F} \bullet \mathbf{n} d\mathbf{S} = \int\int_D \mathbf{F}(r(u, v)) \bullet r_u \times r_v \, dA$.
For nice functions and regions and boundaries: Green's Theorem $\int_{C=\partial S} Pdx + Qdy = \int\int_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$; Stoke's Theorem $\int_{C=\partial S} Pdx + Qdy + Rdz = \int_C \mathbf{F} \bullet d\mathbf{r} = \int\int_S Curl(\mathbf{F}) \bullet \mathbf{n}\, dS = \int\int_C Curl(\mathbf{F})(r(u, v)) \bullet r_u \times r_v \, dA$; Divergence Theorem $\int\int_{S=\partial E} \mathbf{F} \bullet d\mathbf{r}$
$= \int\int\int_E Div(\mathbf{F})(r(u, v))\, dV$.