SHORT REVIEW FOR MATH 231 FINAL

**Geometry:** dot products, cross products, scalar triple product; interpretation as area of parallelogram, volume of parallelepiped.

Euclidean/Cartesian, Polar, Cylindrical, Spherical Coordinates and their volume elements $dxdydz, rdrd\theta, \rho^2 \sin(\phi)d\rho d\phi d\theta$.

Parametric equations of curves $C : r(t)$: (1) line through $P$ in direction $v$ [$P + tv$], (2) line segment from $P$ to $Q$ $[(1-t)P + tQ]$. Tangent line to curve at $P$. Level curves $f(x, y) = k$.

Parametric equations of surfaces $S : R(u, v) = x(u, v), y(u, v), z(u, v)$: (1) plane through $P_0$ with normal vector $n [(P - P_0) \cdot n = 0]$, (2) sphere $\rho = a$ in spherical, (3) cylinder $r = k$ in cylindrical, (4) graph $z = g(x, y)$ or $y = g(x, z)$.

Tangent plane to surface at $P_0 = r(u_0, v_0)$: $n = r_u \times r_v$; graph $z = g(x, y)$ has $n = (-g_x, -g_y, 1)$.

Level surfaces $f(x, y, z) = k$.

Area of surface $A(S) = \iint_S 1dS$; surface integral of function $\iint_S f(x, y, z)dS = \iint_D f(r(u, v))r_u \times r_v dA$.

**Functions:** vector-valued $r(t)$, tangent vector, $r'(t)$; scalar-valued functions, partial derivatives, directional derivatives $D_u(f)$, gradient; differentiable functions, directional derivatives given by gradient dot $u$; Clairaut’s Theorem (mixed partials agree if continuous). Chain Rule. Gradient as direction of maximum increase of function, with $\| \nabla (f)(\vec{x}) \|$ the maximum rate of increase at $\vec{x}$; tangent plane to $z = f(x, y)$ or to implicitly to a level surface $f(x, y, z) = k$.

Local extrema occur at critical points or on the boundary [No Lagrange Multipliers this test!!; a continuous function on a closed bounded set attains its absolute maximum and minimum on that set.

**Integration:** Integration in Euclidean, polar, cylindrical, spherical coordinates. Change of variables, transformations, Jacobians. Change of iteration. Line integrals, work. Surface integrals.

**Regions:** R open [all points $P$ in $R$ have ball $B_\epsilon(P)$ contained in $R$; usually $<$ inequalities], connected [there is a continuous curve from $P$ to $Q$ for any two points in $R$], simply connected [connected and any closed path in $R$ can be continuously shrunk in $R$ to a point, i.e. no holes], closed [all limit points of sequences in $R$ stay in $R$; usually $\leq$ inequalities].

Examples: open/closed disk, annulus, sphere, ball.

**Vector Fields:** $F$, gradients $F = Grad(f) = \nabla f$, curl $curl(F) = \nabla \times F$, divergence $\nabla \cdot F$,

$Curl(Grad(f)) = 0$, Div($Curl(F)$) = 0. Conservative ($F = Grad(f)$) iff mixed partials agree [when $R$ open, simply connected], iff (closed line integrals zero) iff (line integrals independent of path) [when $R$ open, connected].

Oriented surface [smooth choice of normal vector], surface integral of vector field $\int \int_S F \bullet n dS = \int \int_D F(r(u, v)) \bullet r_u \times r_v dA$.

For nice functions and regions and boundaries: Green’s Theorem $\int_{C=\partial S} Pdx + Qdy = \int \int_S (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dA$; Stokes’ Theorem $\int_{C=\partial S} Pdx + Qdy + Rdz = \int_C F \bullet dr = \int \int_S Curl(F) \bullet n dS = \int \int_C Curl(F)(r(u, v)) \bullet r_u \times r_v dA$; Divergence Theorem $\int \int_{S=\partial E} F \bullet dr = \int \int \int_E Div(F)(r(u, v)) dV$. 