Lecture 10 – Parametric Equations

Note Title

Today we look at ways to label points on a curve. We just have to simultaneously name the x & y values.

Use a parameter, a way to name each point on a curve. Usually name this t.

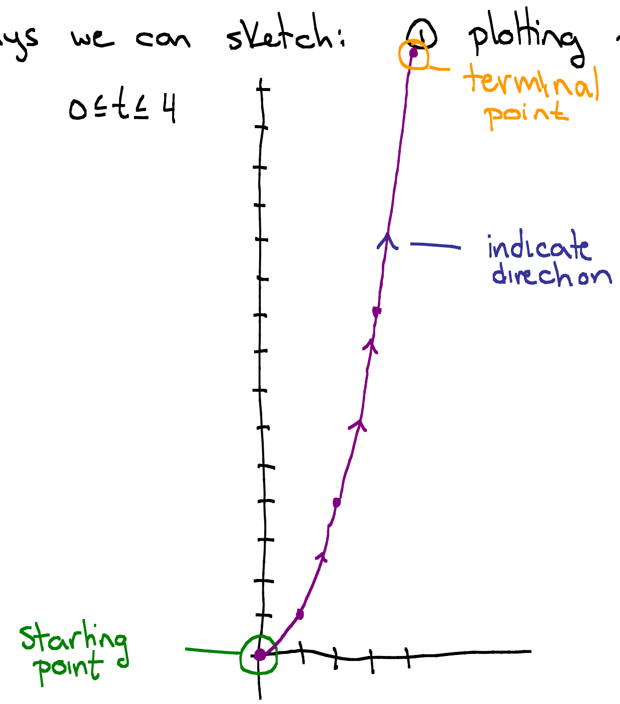
A parametric equation is a pair of equations

$$x = f(t)$$
$$y = g(t)$$

Have 2 ways we can sketch: ① plotting points & ② eliminating t.

Ex: $x = t$, $y = t^2$ $\quad 0 \le t \le 4$

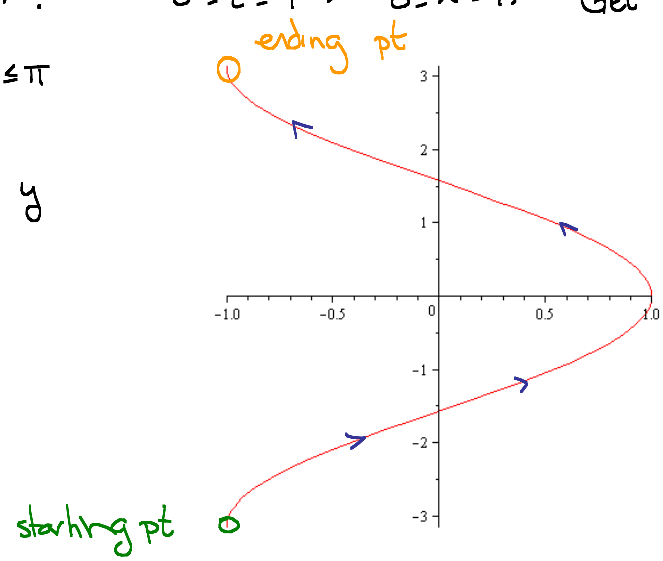
t	x	y
0	0	0
1	1	1
2	2	4
3	3	9
4	4	16



Otherwise: $x = t \Rightarrow y = t^2 = x^2$. $\quad 0 \le t \le 4 \Rightarrow 0 \le x \le 4$. Get same pic.

Ex: $x = \cos t$, $y = t$ $\quad -\pi \le t \le \pi$

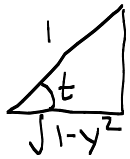
eliminating t: $\quad x = \cos y$



Ex: $x = \cos t$ $\quad 0 \le t \le 2\pi$

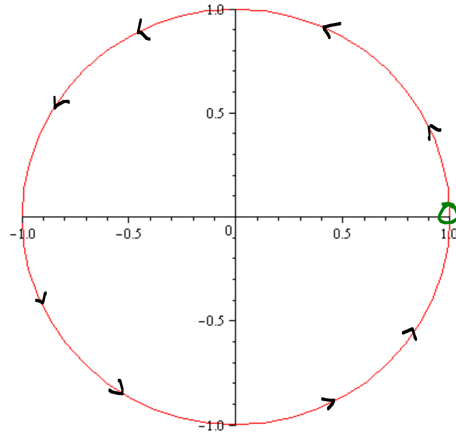
$y = \sin t$

$y = \sin t$:



$\Rightarrow x = \cos t = \pm\sqrt{1-y^2}$

$\Rightarrow x^2 + y^2 = 1$

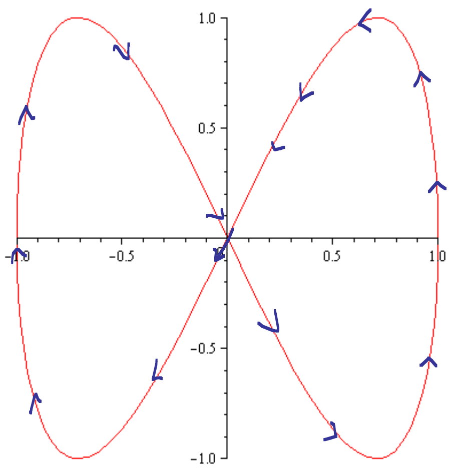


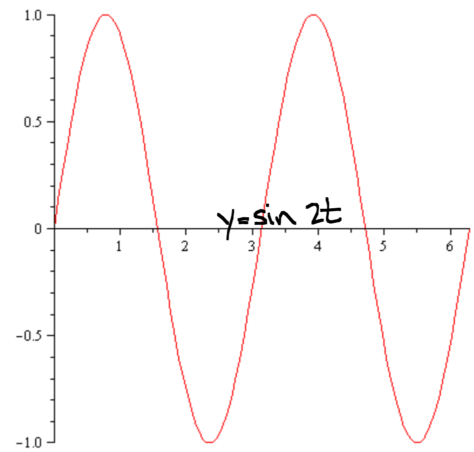
initial & terminal point

Now a bestiary:

$x = \cos t$

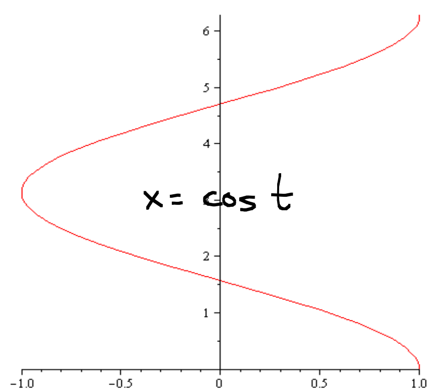
$y = \sin 2t$





y goes $0 \to 1 \to -1 \to 1 \to -1 \to 0$

while

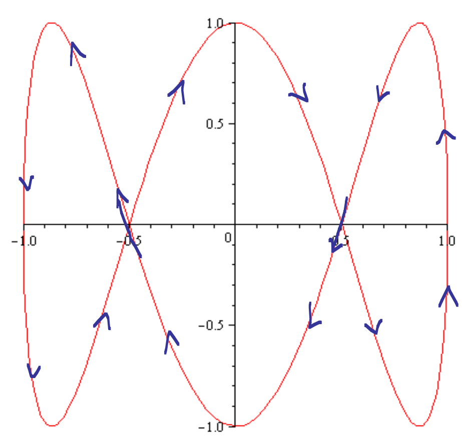


$\leftarrow$ x moves slowly from 1 to -1 then back

$x = \cos t$

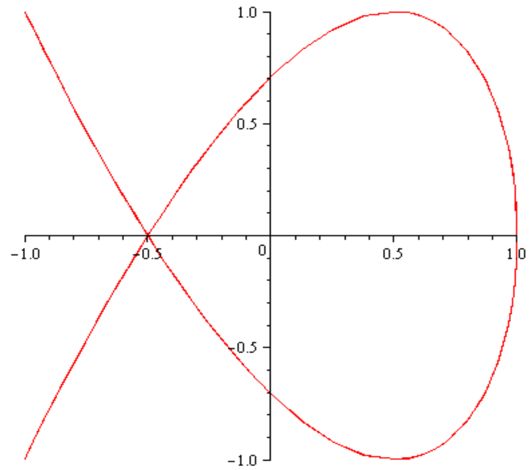
$y = \sin 3t$

$0 \le t \le 2\pi$

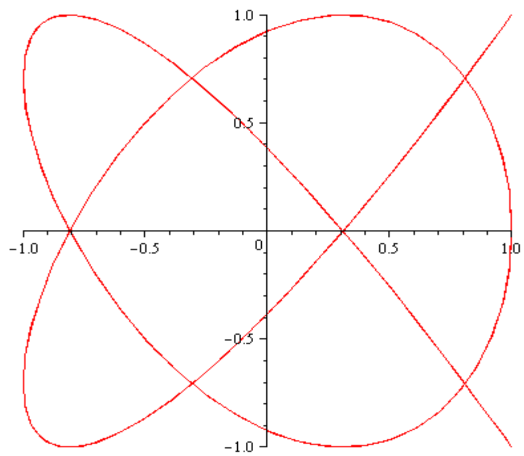


Skipping Ahead:

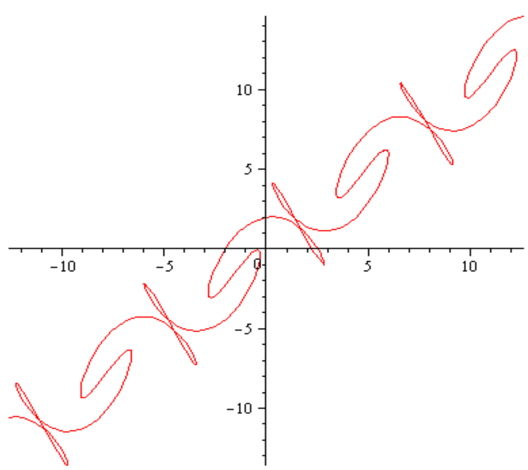
$x = \cos 2t$
$y = \sin 3t$
$0 \le t \le 2\pi$



$x = \cos 4t$
$y = \sin 5t$
$0 \le t \le 2\pi$

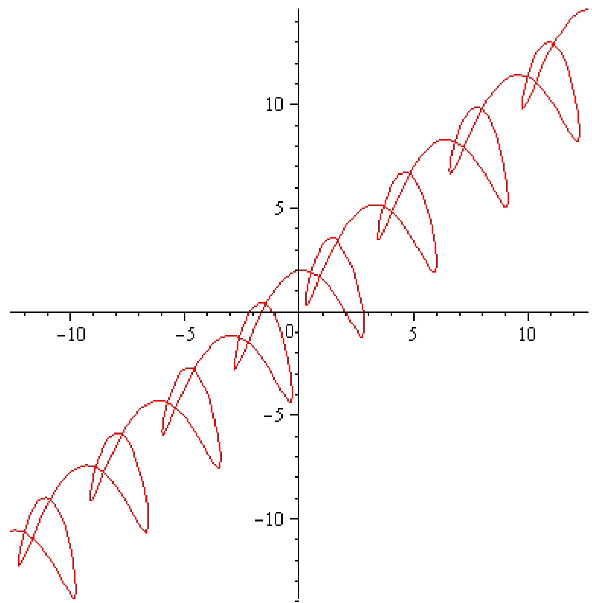


$x = t + 2\sin 2t$
$y = t + 2\cos 3t$
$-4\pi \le t \le 4\pi$

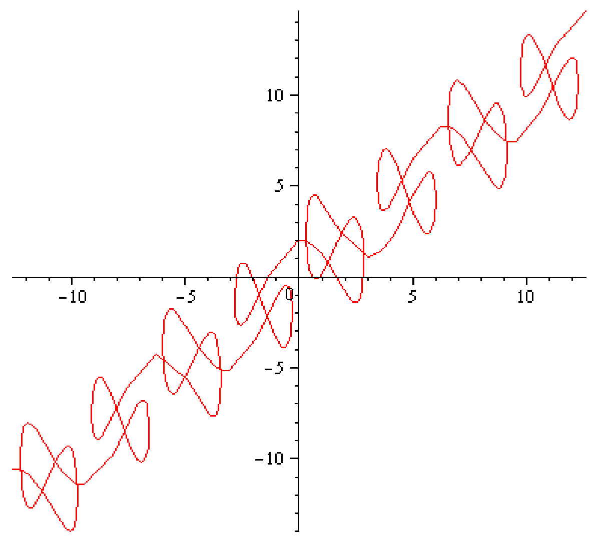


Can see a "phantom" $y = x$.

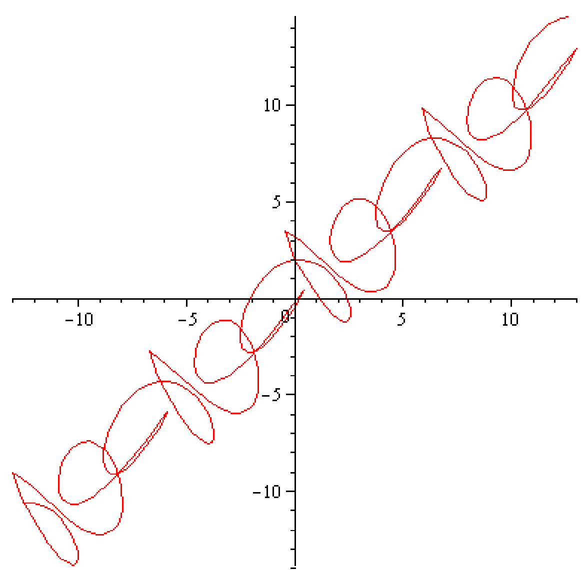
$x = t + 2\sin 2t$
$y = t + 2\cos 4t$
$-4\pi \le t \le 4\pi$



$x = t + 2\sin 2t$
$y = t + 2\cos 5t$
$-4\pi \le t \le 4\pi$

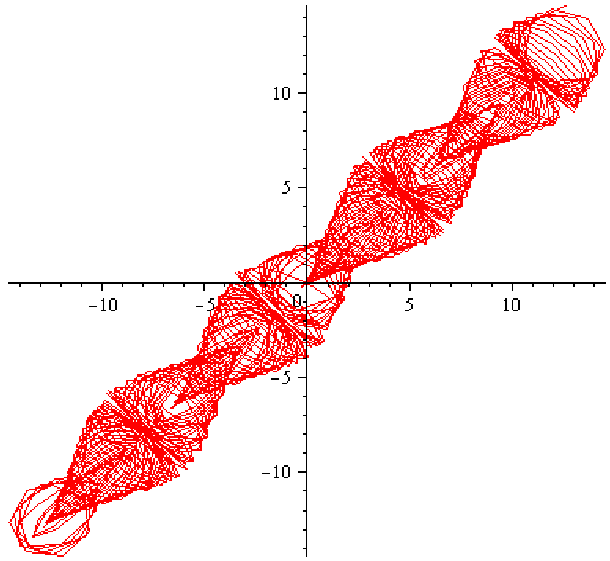


$x = t + 2\sin 3t$
$y = t + 2\cos 4t$
$-4\pi \le t \le 4\pi$



Everyone's favorite

$x = t + 2\sin 55t$
$y = t + 2\cos 54t$
$-4\pi \le t \le 4\pi$



Tangent lines: $y - y_0 = m(x - x_0)$ where $m = \frac{dy}{dx}(x_0, y_0)$

If $x = f(t)$, $y = g(t)$, then $dx = f'(t)\,dt$
$dy = g'(t)\,dt$

So $\frac{dy}{dx} = \frac{g'(t)\,dt}{f'(t)\,dt} = \frac{g'(t)}{f'(t)} = \frac{dy/dt}{dx/dt}$

Ex: $x = \cos t$
$y = t$ find $\frac{dy}{dx}$ @ $t = \pi/2$ $\frac{dx}{dt} = -\sin t$, $\frac{dy}{dt} = 1$

So $\frac{dy}{dx} = \frac{-1}{\sin t}$; @ $t = \pi/2$: $= -1$