Math 132 - Lecture 1: Intro & u Substitution

Note Title

All info needed for the course is on Collab. Key things to note:

① Self-scheduled exams you can retake

② ~~LOTS~~ of homework. All ungraded.

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TODAY: Review of u-substitution.

Idea: Integration undoes differentiation

u-sub " chain rule : $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

u-Sub: $\int f(g(x)) \cdot g'(x)\, dx = \int f(u)\, du, \quad u = g(x)$

Since the left-hand side involves x, after integrating, plug in for u.

Ex: 1) $\int e^{\sin x} \cdot \cos x \, dx$

Let $u = \sin x \Rightarrow du = \cos x \, dx \Rightarrow \int \underbrace{e^{\sin x}}_{e^u} \underbrace{\cos x \, dx}_{du} = \int e^u \, du = e^u + C$

$= \boxed{e^{\sin x} + C}$

2) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$

$u = \cos x \Rightarrow du = -\sin x \, dx \Rightarrow \int \underbrace{\frac{-1}{\cos x}}_{-\frac{1}{u}} \cdot \underbrace{-\sin x \, dx}_{du} = \int -\frac{1}{u} \, du$

$= -\ln|u| + C = \boxed{-\ln|\cos x| + C} = \ln|\sec x| + C$

How do we pick u?

Try to simplify as much as possible: $\sin(x^2+3x+2)$ is hard, $\sin(u)$ is easy.

$\frac{1}{?}$ is hard, $\frac{1}{u}$ is easy

3) $\int 2x \sqrt{x^2+1} \, dx$ hard part

$u = x^2+1$
$du = 2x \, dx$ $\Rightarrow \int \underbrace{\sqrt{x^2+1}}_{\sqrt{u}} \cdot \underbrace{2x \, dx}_{du} = \int \sqrt{u} \, du$

$$= \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{3}(x^2+1)^{3/2} + C}$$

More advanced u-sub: solving for x too. Might have terms <u>not</u> $= u$ or du.

In this case, try to solve $u = g(x)$ for x and plug in.

4) $\int t\sqrt{t-1}\,dt$ $\quad u = t-1 \longleftrightarrow t = u+1$, $du = dt$ $\Rightarrow \int \underbrace{t}_{(u+1)} \cdot \underbrace{\sqrt{t-1}}_{\sqrt{u}}\, \underbrace{dt}_{du} = \int (u+1)\sqrt{u}\,du = \int u^{3/2} + u^{1/2}\,du$

$$= \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{5}(t-1)^{5/2} + \frac{2}{3}(t-1)^{3/2} + C}$$

Last Case: definite integrals.

Fundamental Theorem: If $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Have 2 methods:

1. Use method above, find antiderivative, & plug in.
2. Solve directly:

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

In this case, we forget all about x.

5) $\displaystyle\int_{-2}^{2} x^2 e^{x^3}\,dx$

$u = x^3$
$du = 3x^2\,dx$
$\updownarrow$
$\frac{1}{3}du = x^2\,dx$

$x = -2 \Rightarrow u = (-2)^3 = -8$
$x = 2 \Rightarrow u = (2)^3 = 8$

$$\Rightarrow \int_{-2}^{2} \underbrace{e^{x^3}}_{e^u} \cdot \underbrace{x^2\,dx}_{\frac{1}{3}du} = \int_{-8}^{8} \frac{1}{3}e^u\,du = \frac{1}{3}e^u\Big|_{-8}^{8}$$

$$= \boxed{\frac{1}{3}\left(e^8 - e^{-8}\right)}$$