Math 132 - Lecture 1: Intro & u Substitution

All info needed for the course is on Collab. Key things to note:
1. Self-scheduled exams you can re-take
2. Lots of homework. All ungraded.

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Today: Review of u-substitution.

**Idea:** Integration undoes differentiation

\[ \int f(g(x)) \cdot g'(x) \, dx = \int f(u) \, du, \quad u = g(x) \]

Since the left-hand side involves x, after integrating, plug in for u.

**Ex:** 1) \( \int e^{\sin x} \cdot \cos x \, dx \)

Let \( u = \sin x \) \( \Rightarrow \) \( du = \cos x \, dx \) \( \Rightarrow \)

\[ \int e^{\sin x} \cdot \cos x \, dx = \int e^u \, du = e^u + C \]

\[ \text{Answer: } e^{\sin x} + C \]

2) \( \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx \)

\( u = \cos x \) \( \Rightarrow \) \( du = -\sin x \, dx \) \( \Rightarrow \)

\[ \int \frac{1}{\cos x} \cdot -\sin x \, dx = \int \frac{-1}{u} \, du \]

\[ = -\ln |u| + C = -\ln |\cos x| + C = \ln |\sec x| + C \]

How do we pick \( u \)?

Try to simplify as much as possible: \( \sin(x^2+3x+2) \) is hard, \( \sin(u) \) is easy.

\( \frac{1}{u} \) is hard, \( \frac{1}{u} \) is easy

3) \( \int 2x \cdot \sqrt{x^2+1} \, dx \)

\( u = x^2 + 1 \) \( \Rightarrow \) \( du = 2x \, dx \) \( \Rightarrow \)

\[ \int \sqrt{u} \, du \]

hard part
\[ \frac{2}{3} u^{3/2} + C = \frac{2}{3} \left( \frac{2}{3} \right)^{3/2} + C \]

More advanced \textit{u-sub}: solving for \( x \) too. Might have terms \( \not= u \) or \( du \).

In this case, try to solve \( u = g(x) \) for \( x \) and plug in.

1) \[
\int t\sqrt{t-1} \, dt \quad u = t-1 \quad \Rightarrow \quad t = u+1 \quad \Rightarrow \quad \int (u+1)^{5/2} \, du = \int u^{3/2} \, du + \int u^{5/2} \, du
\]

\[ = \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C = \frac{2}{5} (t-1)^{5/2} + \frac{2}{3} (t-1)^{3/2} + C \]

Last case: definite integrals.

**Fundamental Theorem:** If \( F(x) \) is any antiderivative of \( f(x) \), then

\[ \int_{a}^{b} f(x) \, dx = F(b) - F(a). \]

Have 2 methods:

1. Use method above, find antiderivative, & plug in.
2. Solve directly:

\[ \int_{a}^{b} f(g(x))g'(x) \, dx = \int_{g(a)}^{g(b)} f(u) \, du. \]

In this case, we forget all about \( x \).

3) \[
\int_{-2}^{2} x^2 e^{x^3} \, dx
\]

\[ u = x^3 \quad du = 3x^2 \, dx \quad \Rightarrow u = (-2)^3 - 8 \quad \Rightarrow \quad \int_{-2}^{2} e^{u} \cdot \frac{1}{3} \, du = \frac{1}{3} \int_{-8}^{8} e^{u} \, du = \frac{1}{3} \left( e^{8} - e^{-8} \right) \]