MATH 132 FINAL 12/11/07 3-hour closed-book

NO CALCULATORS; SHOW **ALL** WORK; Put ANSWERS in BOXES

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Name : ______________________ SCORE /300 = ________

Missed p1____ p2 ____ p3____ p4____ p5____ p6____ p7____ p8____ Total missed ________

Problem 1: Find the following definite and indefinite integrals:

(a) $\int_0^1 \ln(1+x^2)\,dx = \boxed{}$

(b) $\int \sin^2(x)\cos^3(x)\,dx = \boxed{} + C$

(c) $\int \frac{dx}{(a^2-x^2)^{3/2}} = \boxed{} + C$

Problem 2: Write the **form** of the partial fraction decomposition of the following rational functions (DO NOT SOLVE FOR THE CONSTANTS):

(a) $\dfrac{2x+3}{x(x^2+2x+1)} =$ $\boxed{}$

(b) $\dfrac{2x+3}{x(x^2+2x+2)} =$ $\boxed{}$

Problem 3: (a) If f is a continuous function on $(-\infty, a]$, then the integral $\int_{-\infty}^{a} f(x)dx$ is **defined** to be convergent if:

(b) Determine whether the improper integral $\displaystyle\int_1^\infty \frac{e^{\sin(x)}}{x^2+3}\,dx$ converges; explain your reasoning, but **do not evaluate** the integral if it converges.

converges $\square$ diverges $\square$

(c) Evaluate the improper integral $\displaystyle\int_1^4 \frac{1}{2x-3}\,dx$ (write DIV if it does not converge). Explain your reasoning.

$$\int_1^4 \frac{1}{2x-3}\,dx = \boxed{}$$

Problem 4: Answer the following questions for the **parameterized curve** $x = e^t, y = e^{-t}$ for $0 \leq t \leq 1$; explain your reasoning.

(a) Set up (but **do not evaluate**) an integral for the length of the curve.

$L = \boxed{}$

(b) Find the equation of the tangent line to the curve at time $t = \frac{1}{2}$.

$y = \boxed{}\, x + \boxed{}$

(c) Find the area between the curve and the x-axis from $t = 0$ to $t = 1$;

$Area = \boxed{}$

(d) Find a Cartesian equation $y = f(x)$ for this curve.

$y = \boxed{}$

Problem 5: (a) Solve the differential equation $y' + y = \dfrac{1}{1+e^x}$ with initial condition $y(0) = 0$.

$y(x) =$ $\boxed{}$

(b) Find the most general solution of $\dfrac{dy}{dx} = 5yx^4$.

$y(x) =$ $\boxed{}$

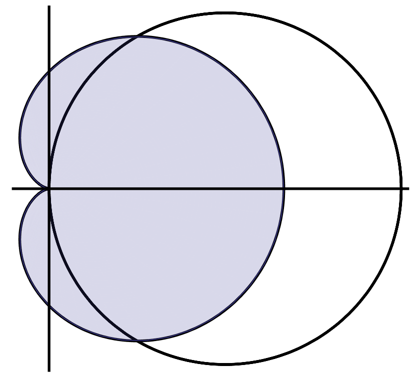
Problem 6: (a) Find the area of the wedge-shaped region inside the polar curve $r = \cos(\theta)$ between $\theta = 0$ and $\theta = \frac{\pi}{4}$.

Area = $\boxed{}$

(b) Give the Cartesian equation of the polar curve $r = \dfrac{1}{\cos(\theta) + \sin(\theta)}$.

$\boxed{}$

(c) For the circle $r = 3\cos(\theta)$ and the cardioid $r = 1 + \cos(\theta)$, set up (but **do not evaluate**) the integral for the area inside the circle but outside the cardioid (the **unshaded** crescent region sketched below):



Area = $\boxed{}$

Problem 7: Write the iterated integral $\int_0^1 \int_0^{3x} f(x,y)dydx$ as an iterated integral with the order of integration interchanged.

Problem 8: In the following 3 parts, state the definitions of convergence, absolute convergence, and conditional convergence of an infinite series $\sum_{n=1}^{\infty} a_n$. The series $\sum_1^{\infty} a_n$ is **defined** to be

(a) **convergent** if:

(b) **absolutely convergent** if:

(c) **conditionally convergent** if

Problem 9: Determine whether the following infinite series converge absolutely, conditionally, or diverge (check one). Mention which tests you use.

(a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{2n^2+1}{n^2+3}\right)$ converges absolutely ☐ converges conditionally ☐ diverges ☐

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\cos(n)}{\sqrt{n}}$ converges absolutely ☐ converges conditionally ☐ diverges ☐

Problem 10: Find **all** values of q such that the following infinite series converge:

(a) $\displaystyle\sum_{n=1}^{\infty} n^q$ converges all q with ☐

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n n^q$ converges for all q with ☐

(c) $\displaystyle\sum_{n=1}^{\infty} q^n$ converges for all q with ☐

(d)Find the **interval of convergence** of $\displaystyle\sum_{n=0}^{\infty} \frac{n!(x+5)^n}{4^n(n+1)!}$. Interval ☐

Problem 11. (a) For a general differentiable function f, the **Taylor series** centered at a is defined to be $\sum_{n=0}^{\infty} c_n(x-a)^n$ where the coefficients are

$$c_n = \boxed{}$$

(b) Find the Taylor series for the function $f(x) = e^x$ centered at $\ln(5)$.

(c) Find the fourth-degree Taylor polynomial $T_4(x)$ centered at 0 for the function $f(x) = \int_0^x e^{-t^2}\,dt$:

$T_4(x) = \sum_{n=0}^{4} c_n x^n$ for coefficients $c_0 = \boxed{}$ $c_1 = \boxed{}$ $c_2 = \boxed{}$ $c_3 = \boxed{}$ $c_4 = \boxed{}$

Problem 12: Identify the following power series with the indicated functions (put the letter of the correct function next to the series):

_____ (i) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$

_____ (ii) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{n}$

_____ (iii) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n+1)!}$

_____ (iv) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$

A: $\arctan(x)$
B: $\arctan(-x)$
C: $-\ln(1+x)$
D: $\frac{1}{e^x}$
E: $-e^x$
F: $\frac{\sin(x)}{x}$
G: $\sin(x)$
H: $\frac{\cos(x)}{2n+1}$

Problem 13: Find the sum of the following series:

(a) $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+2}}{6^{2n+1}(2n)!} = \boxed{}$$

(b) $$-\ln(2) + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \ldots = \boxed{}$$

True-False Questions

Each is worth 4 points; circle the correct answer T or F (you do not need to justify your answer).

1. T F If $\lim_{b\to\infty} \int_{-b}^{b} f(x)dx$ exists, then $\int_{-\infty}^{\infty} f(x)dx$ converges.
2. T F The graph of the parametric curve $x = 2t^{3/5}$, $y = t^{3/5}$ is a straight line.
3. T F If $a_n \geq 0$ for all n and $\sum\limits_{n=1}^{\infty} a_n$ converges, then $\sum\limits_{n=1}^{\infty} (-1)^n a_n$ must converge.
4. T F Every power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ converges for at least one value of x.
5. T F If the series $\sum\limits_{1}^{\infty} c_n x^n$ converges for all $-3 < x < -1$, then $\sum\limits_{1}^{\infty} c_n 2^n$ must converge as well.

PLEDGE IN FULL: