Prop X is second countable $\Rightarrow$ every open cover of X has a countable subcover.

Pf: Let $\{U_i\}_{i\in I}$ be an arbitrary cover of X. Choose a countable basis $\mathcal{B}$. Let $\mathcal{B}' = \{V \in \mathcal{B} \mid \exists i \text{ w/ } V \subseteq U_i\}$. Then $\mathcal{B}'$ is also an open cover of X, since if $x \in X$, then for some i, $x \in U_i$, and since $\mathcal{B}$ is a basis, there is a $V \in \mathcal{B}$ s.t. $x \in V \subseteq U_i \Rightarrow V \in \mathcal{B}'$. Now for each $V \in \mathcal{B}'$, let $i_V \in I$ be such that $V \subseteq U_{i_V}$. Then $\{U_{i_V} \mid V \in \mathcal{B}'\}$ is a countable collection of open sets, and since $\mathcal{B}'$ is a cover of X & $\forall V \in \mathcal{B}'$, $V \subseteq U_{i_V}$, it's a cover. $\square$

Thm (ii), (iii) $\Rightarrow$ (i).

Pf Let $\{U_i\}_{i\in I}$ be an open cover of X. Assume X is sequentially compact, and equivalently totally bounded and complete. Tot. bounded $\Rightarrow$ X is 2^{nd} countable $\Rightarrow$ there is a countable set $J \subseteq I$ s.t. $\{U_j\}_{j\in J}$ is an open cover of X. Choose a bijection $J \longleftrightarrow \mathbb{N}$, so our cover is now indexed on $\mathbb{N}$. For each $n \in \mathbb{N}$, choose an $x_n \in X - \bigcup_{i=1}^{n} U_i$. If $\{U_i\}_{i\in\mathbb{N}}$ has no finite subcover, then this gives a sequence $[x_n]$ in X. Now since $\bigcup_{i=1}^{n} U_i$ is open, for each n, $V_n = X - \bigcup_{i=1}^{n} U_i$ is closed. By assumption, $[x_n]$ has a convergent subsequence $[x_{n_k}]$, with limit x. For all m, $\exists N$ s.t. $n_k > m$ $\forall k \geq N$. In particular, for all m, all but finitely many terms of $[x_{n_k}]$ are in V_m. Since V_m is closed, $x \in V_m$.
$\Rightarrow x \in \bigcap_{i=1}^{\infty} V_i = \bigcap_{i=1}^{\infty}\left(X - \bigcup_{j=1}^{i} U_i\right) = X - \left(\bigcup_{i=1}^{\infty}\bigcup_{j=1}^{i} U_i\right) = X - \left(\bigcup_{i=1}^{\infty} U_i\right) = \emptyset$. This is a contradiction. $\square$