Quiz the Last

Solutions

1. Find the third order Fourier approximation to $f(x) = 3x$.

Need: $\langle 1, f\rangle = \int_{-\pi}^{\pi} 3x\,dx = 0$ $\qquad \langle 1,1\rangle = 2\pi$

$\langle \cos mx, f\rangle = \int_{-\pi}^{\pi} 3x \cos mx = 0$ $\qquad \langle \sin mx, \sin mx\rangle = \pi$

(odd . even = odd)

$$\langle \sin mx, f\rangle = \int_{-\pi}^{\pi} 3x \sin mx\,dx = \left(3x\cdot \frac{-1}{m}\cos mx + \frac{3}{m^2}\sin mx\right)\Big|_{-\pi}^{\pi}$$

$$= \frac{6\pi(-1)^{m+1}}{m}$$

$3x \searrow \sin mx$

$3 \searrow -1/m \cos mx \; +$

$0 \searrow -1/m^2 \sin mx \; -$

$$\Rightarrow 3x \approx \frac{\langle 3x, \sin x\rangle}{\langle \sin x, \sin x\rangle}\sin x + \ldots = \boxed{6\sin x - 3\sin 2x + 2\sin 3x}$$

2. Convert $\{1, x\}$ into an orthogonal set with respect to the inner product $\langle f, g\rangle = \int_0^2 f(x)g(x)dx$.

$\bar{u} = 1$ $\qquad \langle 1, x\rangle = \int_0^2 x\,dx = 2$

$\qquad \langle 1,1\rangle = \int_0^2 1\,dx = 2$

$$\bar{u}_2 = x - \frac{\langle 1, x\rangle}{\langle 1,1\rangle}\cdot 1$$

$$= x - 1$$

$$\rightsquigarrow \boxed{\{1, x-1\}}$$

3. Let $L\colon P_2(x) \to P_1(x)$ be given by $L(p) = p' + 2p''$. Let $\mathcal{B} = \{1, x, x^2\}$ be a basis for P_2, and let $\mathcal{B}' = \{1, x+1\}$ be a basis for P_1. Find the matrix representation of L with respect to $\mathcal{B}$ and $\mathcal{B}'$: ${}_{\mathcal{B}'}[L]_{\mathcal{B}}$.

$L(1) = 0 = 0\cdot 1 + 0\cdot(x+1)$

$L(x) = 1 = 1\cdot 1 + 0\,(x+1)$

$L(x^2) = 2x + 4 = 2\cdot 1 + 2(x+1)$

$$\Rightarrow [L] = \begin{array}{c c} & \begin{array}{ccc} 1 & x & x^2 \end{array} \\ \begin{array}{c} 1 \\ x+1 \end{array} & \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \end{array}$$