Quiz # 6

Solutions

[redacted] = give pts for

April 3, 2008

1. Find a basis for the kernel and range of the linear transformation $L\colon \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$L(\mathbf{v}) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix} \mathbf{v}.$$

this is also fine. If they don't say anything but still solve this, also +1.

+1 [redacted] $= \{ \bar{v} \mid \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix} \bar{v} = \bar{0} \}$ [redacted] +1

$\Rightarrow$ x, z lead, y free $\Rightarrow$ sol: [redacted], so a basis is [redacted].

+1 +1

Range: [redacted] $= \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\leftarrow$ basis vectors

+1

$\Rightarrow$ basis is [redacted] +1

(Note that another basis is $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$, $\leftarrow$ +2 if they do this the columns corresponding to the leading rows in the RRE form for the kernel)

2. Use Gram-Schmidt to turn $\{(1,1,3), (0,3,1), (1,1,0)\}$ into an orthogonal basis (it doesn't have to be orthonormal).

$\bar{w}_1 = (1,1,3)$ +1 [redacted] +1

[redacted] $= (0,3,1) - \frac{6}{11}(1,1,3) =$ [redacted]

+1 +1 $\bar{w}_2 \cdot \bar{w}_2 = \frac{36}{121} + \frac{729}{121} + \frac{49}{121} = \frac{814}{121} = \frac{74}{11}$

[redacted] $\frac{21}{11}$

[redacted]

$= (1,1,0) - \frac{2}{11}(1,1,3) - \frac{21/11}{74/11}\left(-\frac{6}{11}, \frac{27}{11}, \frac{-7}{11}\right)$

$= \left(\frac{9}{11}, \frac{9}{11}, \frac{-6}{11}\right) - \frac{21}{74}\left(\frac{-6}{11}, \frac{27}{11}, \frac{-7}{11}\right)$

$=$ [redacted]

I don't care what numbers they get or if they simplify things.