Quiz #5

Solutions

1. Show that $\{f|f(x) = f(-x)\}$ is a subspace of $C((-1, 1))$.

   1) $0(x) = 0(-x) = 0$ so $0 \in$ set

   2) $f, g \in$ Set $\Rightarrow$ $(f+g)(x) = f(x) + g(x) = f(-x) + g(-x) = (f+g)(-x)$
      $\Rightarrow f+g \in$ Set

   3) $f \in$ Set $\Rightarrow$ $(af)(x) = a(f(x)) = a(f(-x)) = (af)(-x)$
      $\Rightarrow af \in$ Set

   $\Rightarrow$ Set is a subspace

2. Is the set of 2×2 matrices with 1s along the diagonal a subspace? If so, show it. If not, show how at least one required property fails.

   No. 1) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ not in the set

   2) $A, B \in$ Set $\Rightarrow$ $A+B$ <u>not</u> in Set

   3) $A \in$ Set $\Rightarrow$ aA <u>not</u> in Set $(a \neq 1)$

3. Is the function $x + 5$ in the space spanned by $x + 1$ and $x + 3$?

   $a(x+1) + b(x+3) = x + 5$
   $\Updownarrow$

   $$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 3 & 5 \end{array}\right] \longleftrightarrow \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & 4 \end{array}\right] \longleftrightarrow \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right] \Rightarrow \text{in span}$$

   (since $\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 2$, $\{x+1, x+3\}$ is a basis of $P_1(x)$)

4. Show that $\{(1, 1, 1), (0, 1, 2), (3, 0, 1)\}$ is a basis for $\mathbb{R}^3$.

   Basis iff $$\begin{vmatrix} 1 & 0 & 3 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = (1+6+0) - (3+0+0) = 4 \neq 0. \checkmark$$