Quiz #3

Name: Solutions

1. Find the inverse of the following 3×3 matrix using Gauss-Jordan elimination

$$\begin{bmatrix} 1 & -3 & 5 \\ 2 & -5 & 7 \\ 3 & -7 & 10 \end{bmatrix}$$

$$-2, -3 \quad \left[\begin{array}{ccc|ccc} 1 & -3 & 5 & 1 & 0 & 0 \\ 2 & -5 & 7 & 0 & 1 & 0 \\ 3 & -7 & 10 & 0 & 0 & 1 \end{array}\right] -3 \Rightarrow \left[\begin{array}{ccc|ccc} 1 & -3 & 5 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 2 & -5 & -3 & 0 & 1 \end{array}\right] -2 \Rightarrow \left[\begin{array}{ccc|ccc} 1 & -3 & 5 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] 3, -5$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -3 & 0 & -4 & 10 & -5 \\ 0 & 1 & 0 & 1 & -5 & 3 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] 3 \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -5 & 4 \\ 0 & 1 & 0 & 1 & -5 & 3 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} -1 & -5 & 4 \\ 1 & -5 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

2. Solve the following system using the method of LU decomposition

$$\begin{aligned} x + 2y + 4z &= 1 \\ 2x + 5y + 10z &= 3 \\ 3x + 8y + 17z &= 6 \end{aligned}$$

$$-3 \quad \overset{A}{\begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 10 \\ 3 & 8 & 17 \end{bmatrix}} -2 \Rightarrow \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 0 & 2 & 5 \end{bmatrix} -2 \Rightarrow \overset{U}{\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$U = \begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ -2 & 1 & \\ -3 & 0 & 1 \end{bmatrix} A \Rightarrow A = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}$$

1) $L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \Rightarrow \begin{aligned} x &= 1 \\ 2x + y &= 3 \\ 3x + 2y + z &= 6 \end{aligned} \Rightarrow x = y = z = 1$

2) $U \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Leftrightarrow \begin{aligned} x + 2y + 4z &= 1 \\ y + 2z &= 1 \\ z &= 1 \end{aligned}$

$\Rightarrow z = 1, \; y = -1, \; x = -1$

So $A\bar{x} = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \Rightarrow \bar{x} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$.