HW 6

Exercise Set 2.6

$\boxed{2.}$ $T((x_1,y_1)+(x_2,y_2)) = T(x_1+x_2, y_1+y_2)$
$= (3(x_1+x_2)+y_1+y_2,\ 2(y_1+y_2),\ x_1+x_2-y_1-y_2)$
$= (3x_1+3x_2+y_1+y_2,\ 2y_1+2y_2,\ x_1+x_2-y_1-y_2)$
$= (3x_1+y_1,\ 2y_1,\ x_1-y_1) + (3x_2+y_2,\ 2y_2,\ x_2-y_2) = T(x_1,y_1) + T(x_2,y_2)$,
and $T(c(x,y)) = T(cx,cy) = (3cx+cy,\ 2cy,\ cx-cy) = c(3x+y,\ 2y,\ x-y) = cT(x,y)$.
Thus T is linear. $T(1,2) = (5,4,-1)$ and $T(2,-5) = (1,-10,7)$.

4. (a) $T(c(x,y,z)) = T(cx,cy,cz) = (3cx,(cy)^2) = (3cx,c^2y^2) = c(3x,cy^2) \neq c(3x,y^2)$.

Thus scalar multiplication not preserved. T is not linear.

(b) $T(c(x,y,z)) = T(cx,cy,cz) = (cx+2,4cy) \neq c(x+2,4y) = cT(x,y,z)$, so T is not linear.

13. (a) $\begin{bmatrix}1\\0\end{bmatrix} \mapsto \begin{bmatrix}2\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix} \mapsto \begin{bmatrix}-5\\3\end{bmatrix}$, so $A = \begin{bmatrix}2 & -5\\0 & 3\end{bmatrix}$.

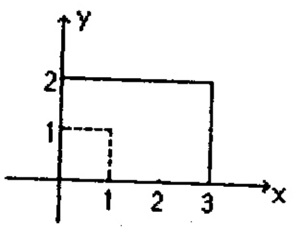
(b) $\begin{bmatrix}1\\0\end{bmatrix} \mapsto \begin{bmatrix}0\\-3\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix} \mapsto \begin{bmatrix}2\\0\end{bmatrix}$, so $A = \begin{bmatrix}0 & 2\\-3 & 0\end{bmatrix}$.

18. $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}\frac{x+y}{2}\\ \frac{x+y}{2}\end{bmatrix}$, so $\begin{bmatrix}1\\0\end{bmatrix} \mapsto \begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix} \mapsto \begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}$, and $A = \begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$.

$\begin{bmatrix}4\\2\end{bmatrix} \mapsto \begin{bmatrix}3\\3\end{bmatrix}$.

19. $\begin{bmatrix}1\\0\end{bmatrix} \mapsto \begin{bmatrix}a\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix} \mapsto \begin{bmatrix}0\\b\end{bmatrix}$, so $A = \begin{bmatrix}a & 0\\0 & b\end{bmatrix}$.

If a = 3 and b = 2, the unit square becomes a 3x2 rectangle.



Exercise Set 2.8

1. [redacted]

(b) The elements in column 2 are not numbers between zero and 1, so the matrix is not stochastic.

[redacted]

[redacted]

(f) There is a negative number in column 3, so the matrix is not stochastic.

2. $\begin{bmatrix} x & y \\ 1-x & 1-y \end{bmatrix}\begin{bmatrix} u & w \\ 1-u & 1-w \end{bmatrix} = \begin{bmatrix} xu+y(1-u) & xw+y(1-w) \\ (1-x)u+(1-y)(1-u) & (1-x)w+(1-y)(1-w) \end{bmatrix}$.

The elements of the product are all greater than or equal to zero and the sum of the terms in each column is 1, so the matrix is stochastic. Also, assuming $y \le x$, $xu+y(1-u) \le xu+x(1-u) = x \le 1$. Assuming $y > x$, $xu+y(1-u) < yu+y(1-u) = y \le 1$. So the first element is always less than or equal to 1. Similar argument holds for other elements

6. 2008: $\begin{bmatrix} .99 & .02 \\ .01 & .98 \end{bmatrix}\begin{bmatrix} 245 \\ 52 \end{bmatrix} = \begin{bmatrix} 243.59 \\ 53.41 \end{bmatrix}$. $\quad$ *Metro*: 243.59 *million*, *Nonmetro*: 53.41 *million*

2009: $\begin{bmatrix} .99 & .02 \\ .01 & .98 \end{bmatrix}\begin{bmatrix} 243.59 \\ 53.41 \end{bmatrix} = \begin{bmatrix} 242.2223 \\ 54.7777 \end{bmatrix}$. $\quad$ *Metro*: 242.2223 *million*, *Nonmetro*: 54.7777 *million*

2010: $\begin{bmatrix} .99 & .02 \\ .01 & .98 \end{bmatrix}\begin{bmatrix} 242.2223 \\ 54..7777 \end{bmatrix} = \begin{bmatrix} 240.8956 \\ 56.1044 \end{bmatrix}$. $\quad$ *Metro*: 240.8956 *million*, *Nonmetro*: 56.1044 *million*

$\begin{bmatrix} .99 & .02 \\ .01 & .98 \end{bmatrix}^3 = \begin{bmatrix} .9709 & .0582 \\ .0291 & .9418 \end{bmatrix}$; the probability that a person living in a metropolitan area in 2007 will still be living in a metropolitan area in 2010 is .9709.

7. 2008: $\begin{bmatrix} .96 & .01 & .015 \\ .03 & .98 & .005 \\ .01 & .01 & .98 \end{bmatrix}\begin{bmatrix} 82 \\ 163 \\ 52 \end{bmatrix} = \begin{bmatrix} 81.13 \\ 162.46 \\ 53.41 \end{bmatrix}$.

City: 81.13 million, Suburb: 162.46 million, Nonmetro: 53.41 million

2009: $\begin{bmatrix} .96 & .01 & .015 \\ .03 & .98 & .005 \\ .01 & .01 & .98 \end{bmatrix}\begin{bmatrix} 81.13 \\ 162.46 \\ 53.41 \end{bmatrix} = \begin{bmatrix} 80.3106 \\ 161.9118 \\ 54.7777 \end{bmatrix}$.

City: 80.3106 million, Suburb: 161.9118 million, Nonmetro: 54.7777 million

2010: $\begin{bmatrix} .96 & .01 & .015 \\ .03 & .98 & .005 \\ .01 & .01 & .98 \end{bmatrix}\begin{bmatrix} 80.3106 \\ 161.9118 \\ 54.7777 \end{bmatrix} = \begin{bmatrix} 79.5389 \\ 161.3567 \\ 56.1044 \end{bmatrix}$.

City: 79.5389 million, Suburb: 161.3567 million, Nonmetro: 56.1044 million

$\begin{bmatrix} .96 & .01 & .015 \\ .03 & .98 & .005 \\ .01 & .01 & .98 \end{bmatrix}^2 = \begin{bmatrix} .9221 & .0196 & .0292 \\ .0583 & .9608 & .0103 \\ .0197 & .0197 & .9606 \end{bmatrix}$, so the probability that a person living in the city in 2007 will be living in a nonmetropolitan area in 2009 is .0197.

12. (a) .6 $\quad$ (b) next generation: $\begin{bmatrix} 1 & .4 \\ 0 & .6 \end{bmatrix}\begin{bmatrix} 10000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 10400 \\ 600 \end{bmatrix}$ $\begin{matrix}\text{nonfarming} \\ \text{farming}\end{matrix}$

$\begin{bmatrix} 1 & .4 \\ 0 & .6 \end{bmatrix}^4\begin{bmatrix} 10000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 10870.4 \\ 129.6 \end{bmatrix}$, so the distribution after four generations will be 10870 nonfarming and 130 farming.

(c) $\begin{bmatrix} 1 & .4 \\ 0 & .6 \end{bmatrix}^2 = \begin{bmatrix} 1 & .64 \\ 0 & .36 \end{bmatrix}$, so the probability the grandson will be a farmer is .36.

1.

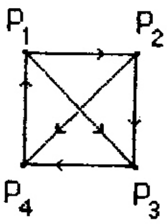
(b) adjacency matrix
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
distance matrix
$$\begin{bmatrix} 0 & 1 & 2 & 3 & 3 \\ 1 & 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 1 & 1 \\ 2 & 1 & 2 & 0 & 3 \\ x & x & x & x & 0 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ x & 0 & x & x & x \\ x & x & 0 & x & x \\ x & x & x & 0 & x \\ x & x & x & x & 0 \end{bmatrix}$$

(d) [redacted]
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
[redacted]
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ x & 0 & 1 & 2 & 1 \\ x & 1 & 0 & 1 & 2 \\ x & x & x & 0 & 1 \\ x & x & x & x & 0 \end{bmatrix}$$

2. (a) 2 (b) undefined (c) undefined (d) undefined

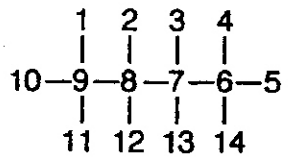
3. [redacted] (b)



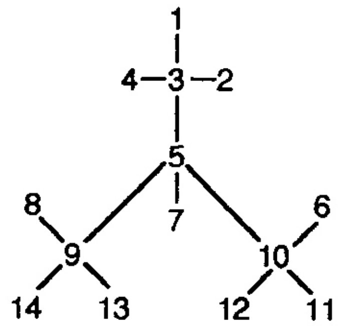
[redacted]

[redacted]

6.



Butane

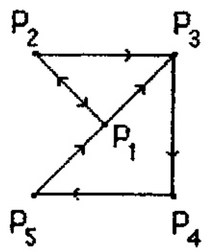


Isobutane

$$\begin{bmatrix} 0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&1&0&1&0&0&0&0&0&0&1\\ 0&0&1&0&0&1&0&1&0&0&0&0&1&0\\ 0&1&0&0&0&0&1&0&1&0&0&1&0&0\\ 1&0&0&0&0&0&0&1&0&1&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0 \end{bmatrix} \qquad \begin{bmatrix} 0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 1&1&0&1&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&1&0&1&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&1&0&0&0&0&1&1\\ 0&0&0&0&1&1&0&0&0&0&1&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0 \end{bmatrix}$$

Yes, the adjacency matrix of any graph is symmetric.

7.



(a) $P_2 \to P_3 \to P_4 \to P_5$

length = 3

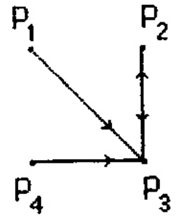
(b) $P_3 \to P_4 \to P_5 \to P_1 \to P_2$

length = 4

distance matrix

$$\begin{bmatrix} 0&1&1&2&3\\ 1&0&1&2&3\\ 3&4&0&1&2\\ 2&3&3&0&1\\ 1&2&2&3&0 \end{bmatrix}$$

(b)



8).

(b) $(a_{21})^2 = 0$. There is no 2-path from P_2 to P_1.
$(a_{33})^2 = 1$. There is one 2-path from P_3 to P_3.
$(a_{42})^2 = 1$. There is one 2-path from P_4 to P_2.
$(a_{13})^3 = 1$. There is one 3-path from P_1 to P_3.
$(a_{32})^3 = 1$. There is one 3-path from P_3 to P_2.
$(a_{34})^3 = 0$. There is no 3-path from P_3 to P_4.

12. There is a 2-path from P_1 to P_3 and a 2-path from P_1 to P_4. The possible 2-paths are

$P_1 \to P_2 \to P_3$ or $P_1 \to P_4 \to P_3$

$P_1 \to P_2 \to P_4$ or $P_1 \to P_3 \to P_4$

The only combination of these that does not produce additional 2-paths is $P_1 \to P_2 \to P_3$ and $P_1 \to P_2 \to P_4$. The dashed lines in the digraph indicate arcs that do not cause additional 2-paths and therefore may be in the digraph.