SECTION 2.4

11. We solve the three sets of equations simultaneously:

$$\begin{bmatrix} 1/9 & 3/9 & 5/9 \\ 3/9 & 0 & -3/9 \\ -2/9 & 3/9 & -1/9 \end{bmatrix}\begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 2 \\ 3 & 4 & -3 \end{bmatrix} = \begin{bmatrix} 22/9 & 23/9 & -4/9 \\ -6/9 & -12/9 & 24/9 \\ 1/9 & -1/9 & -1/9 \end{bmatrix}.$$

Thus the solutions are, in turn,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22/9 \\ -6/9 \\ 1/9 \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 23/9 \\ -12/9 \\ -1/9 \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4/9 \\ 24/9 \\ -1/9 \end{bmatrix}.$$

32. T_1, -3row1:

$$E_1 = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \text{ Check: } \begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} -3a & -3b & -3c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

T_2, row1 + 3row2:

$$E_2 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \text{ Check: } \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a+3d & b+3e & c+3f \\ d & e & f \\ g & h & i \end{bmatrix}.$$

33. √ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ interchanges rows 1 and 2. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

√ (d) $\begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ multiplies row 1 by 1/3. $\begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

√ (f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ adds 3 times row 2 to row 3.

The inverse adds -3 times row 2 to row 3. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$.

In general, to find the inverse of an elementary matrix E:
Swapping rows: $E^{-1} = E$.
Multiply row k by c: Elementary matrix E has $e_{kk} = c$. E^{-1} has $e_{kk} = 1/c$.
Add c times row i to row k: Element e_{ki} of E is c. Element e_{ki} of E^{-1} is -c.
(Other elements of E and E^{-1} are the same.) √

37. √

R	E	T	R	E	A	T
18	5	20	18	5	1	20

The vectors are $\begin{bmatrix} 18 \\ 5 \end{bmatrix}, \begin{bmatrix} 20 \\ 18 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 20 \\ 27 \end{bmatrix}$. Get

$$\begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 18 & 20 & 5 & 20 \\ 5 & 18 & 1 & 27 \end{bmatrix} = \begin{bmatrix} 57 & 26 & 17 & -1 \\ 44 & 24 & 13 & 6 \end{bmatrix}$$

Coded message is 57, 44, 26, 24, 17, 13, -1, 6.

39. The decoding matrix is $\begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix}$.

$\begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 49 & -5 & -61 \\ 38 & -3 & -39 \end{bmatrix} = \begin{bmatrix} 16 & 1 & 5 \\ 5 & 3 & 27 \end{bmatrix}$, and the message is PEACE.

SECTION 2.5

5. (b) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $\quad A\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(g) $A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ $\quad A\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{\sqrt{3}}{2} \\ -\sqrt{3} + \frac{1}{2} \end{bmatrix}$

11. (a) $T_2 \circ T_1(\mathbf{x}) = A_2 A_1 \mathbf{x} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\mathbf{x} = \begin{bmatrix} 5 & 5 \\ -4 & -4 \end{bmatrix}\mathbf{x}$, so

$T_2 \circ T_1\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 5 & 5 \\ -4 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 20 \\ -16 \end{bmatrix}$.

21. (b) $d(P, Q) = \|u - v\| = \|A(u-v)\|$
$= \|Au - Av\| = d(S, T)$

21. (a) It was proved in the text that $\mathbf{u}\cdot\mathbf{v} = A\mathbf{u}\cdot A\mathbf{v}$ and $\|\mathbf{u}\| = \|A\mathbf{u}\|$. Thus

$\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{A\mathbf{u}\cdot A\mathbf{v}}{\|A\mathbf{u}\|\,\|A\mathbf{v}\|}$, and so the angle between the vectors **u** and **v** and the angle between the vectors $A\mathbf{u}$ and $A\mathbf{v}$ are the same.

24. (b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 7 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

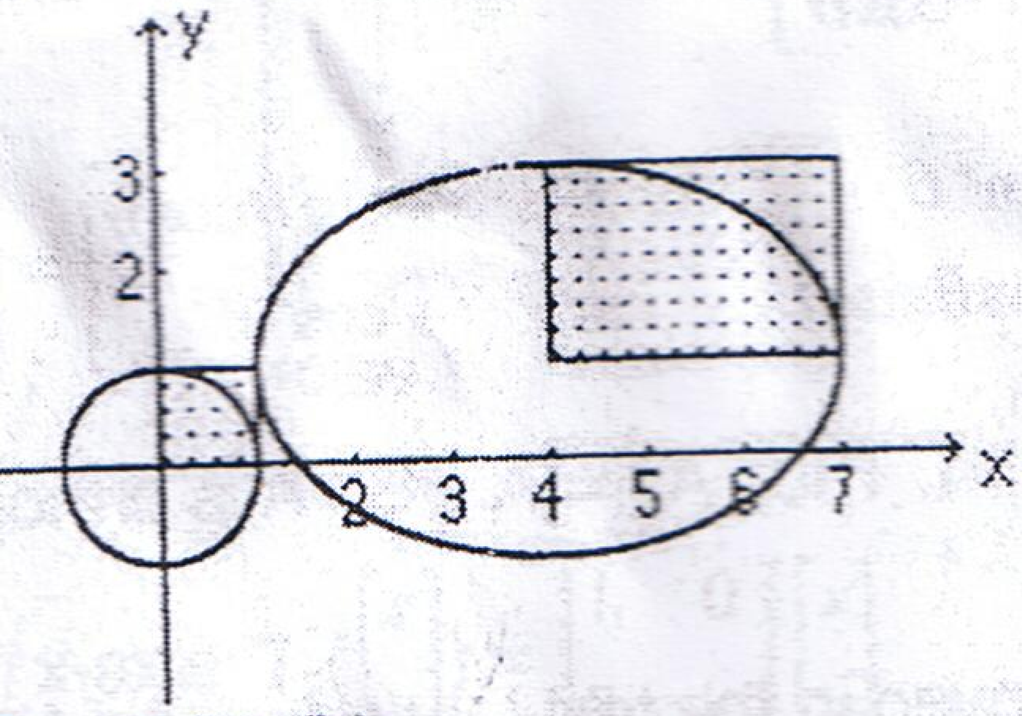

Figure for (b)

$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 3x+4 \\ 2y+1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$, so the image of $x^2 + y^2 = 1$ is $\frac{(x-4)^2}{9} + \frac{(y-1)^2}{4} = 1$.

SECTION 7.2

10. The matrix equation $A\mathbf{x} = \mathbf{b}$ is $\begin{bmatrix} 2 & 1 & 0 \\ 6 & 4 & -1 \\ 2 & 0 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 25 \\ 20 \end{bmatrix}$.

$$\begin{bmatrix} 2 & 1 & 0 \\ 6 & 4 & -1 \\ 2 & 0 & 4 \end{bmatrix} \underset{\substack{R2+(-3)R1 \\ R3+(-1)R1}}{\approx} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 4 \end{bmatrix} \underset{R3+R2}{\approx} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} = U, \text{ and}$$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$. $L\mathbf{y} = \begin{bmatrix} 9 \\ 25 \\ 20 \end{bmatrix}$ gives $\mathbf{y} = \begin{bmatrix} 9 \\ -2 \\ 9 \end{bmatrix}$, and $U\mathbf{x} = \mathbf{y}$ gives $\mathbf{x} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$.

24. $\begin{bmatrix} 6 & -2 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ 0 & 12 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 0 & 6 \end{bmatrix}$.

25. If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = LU = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \begin{bmatrix} p & q & r \\ 0 & s & t \\ 0 & 0 & u \end{bmatrix}$, then $ap = 1$, so $a \neq 0$ and $p \neq 0$.

$aq = 0$ and $ar = 0$, so $q = r = 0$. Also $bp = 0$ and $dp = 0$, so $b = d = 0$. Thus

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & c & 0 \\ 0 & e & f \end{bmatrix} \begin{bmatrix} p & 0 & 0 \\ 0 & s & t \\ 0 & 0 & u \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cs & ct \\ 0 & es & et+fu \end{bmatrix}$. $cs = 0$, so one of c and s must be zero, but $ct = es = 1$, so neither c nor s can be zero. Since no number can be both zero and nonzero, we must conclude that the matrices L and U do not exist.