21.

(b) Submatrix products: $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 6 & 12 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ 1 & 3 \end{bmatrix}$.

$AB = \begin{bmatrix} 1 & -1 \\ 7 & 15 \end{bmatrix}$.

23 (a), (c) $A = \left[\begin{array}{ccc|c} 2 & 0 & 3 & -1 \\ \hline 6 & 2 & -5 & 9 \\ \hline 1 & 1 & 1 & 1 \end{array}\right]$, or $\left[\begin{array}{ccc|c} 2 & 0 & 3 & -1 \\ \hline 6 & 2 & -5 & 9 \\ 1 & 1 & 1 & 1 \end{array}\right]$, $\left[\begin{array}{ccc|c} 2 & 0 & 3 & -1 \\ 6 & 2 & -5 & 9 \\ \hline 1 & 1 & 1 & 1 \end{array}\right]$, $\left[\begin{array}{ccc|c} 2 & 0 & 3 & -1 \\ 6 & 2 & -5 & 9 \\ 1 & 1 & 1 & 1 \end{array}\right]$

Section 2.2

33. Since X_1 is a solution then the scalar multiple of X_1 by a is a solution. aX_1 is a solution. Similarly bX_2 is a solution. Since the sum of two solutions is a solution $aX_1 + bX_2$ is a solution.

$AX_1 = 0 \Rightarrow aAX_1 = 0$
$AX_2 = 0 \Rightarrow bAX_2 = 0$ $\Rightarrow aAX_1 + bAX_2 = 0 \Rightarrow A(aX_1 + bX_2) = 0$

42. Consider the system $AX = B$. Let $A_1, \ldots, A_n$ be the columns of A. Suppose B is a linear combination of $A_1, \ldots, A_n$. Let $B = x_1A_1 + \ldots + x_nA_n$. This can be written $B = AX_0$ where $X_0 = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ (see Section 2.1). This value of X_0 is a solution to the system.

45. (a) Transmission matrix = ~~[illegible]~~.

$\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$

(b) Outputs ~~[illegible]~~ ; ~~V=1 volts, I=6 amps.~~

$\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$; $V = 0$ volts $I = 2$ amps

Section 2.3

27. (b) $G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

$g_{12} = 0, g_{13} = 1, g_{14} = 0,$
$g_{23} = 1, g_{24} = 1, g_{34} = 0,$
so $1 \to 3 \to 2 \to 4$ or $4 \to 2 \to 3 \to 1$.

$P = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

$p_{12} = 1, p_{13} = 1, p_{23} = 0,$
so $2 \to 1 \to 3$ or $3 \to 1 \to 2$.

(d) $G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$.

$g_{12} = 0, g_{13} = 1, g_{14} = 0,$
$g_{23} = 0, g_{24} = 1, g_{34} = 1,$
so $1 \to 3 \to 4 \to 2$ or $2 \to 4 \to 3 \to 1$.

$P = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

$p_{12} = 1, p_{13} = 1, p_{14} = 0,$
$p_{23} = 0, p_{24} = 0, p_{34} = 1,$
so $2 \to 1 \to 3 \to 4$ or $4 \to 3 \to 1 \to 2$.

31. (a) $f_{ij} = a_{i1}a_{j1} + a_{i2}a_{j2} + \ldots + a_{in}a_{jn}$ as in the graves model. $a_{ik}a_{jk}$ is 1 if both person i and person j are friends of person k and is zero otherwise. So the sum of these terms is the number of friends person i and person j have in common.

(b) The matrix A is symmetric when friendships are mutual and is not symmetric when friendships are not mutual. Define a_{ij} to be 1 if person i considers person j to be his friend and zero otherwise. f_{ij} will then be the number of people both person i and person j consider to be their friends.