3. (a) (a, a, b) + (c, c, d) = (a+c, a+c, b+d). 1st components same. Closed under addition. k(a, a, b) = (ka, ka, kb). 1st components same. Closed under scalar multiplication. Subspace. (a, a, b) = a(1, 1, 0) + b(0, 0, 1). Vectors (1, 1, 0) and (0, 0, 1) span space and are linearly independent. {(1, 1, 0), (0, 0, 1)} is a basis. Dimension is 2.

(b) (a, 2a, b) + (c, 2c, d) = (a+c, 2(a+c), b+d). 2nd component is twice first. Closed under addition. k(a, 2a, b) = (ka, 2ka, kb). 2nd component is twice 1st. Closed under scalar multiplication. Subspace. (a, 2a, b) = a(1, 2, 0) + b(0, 0, 1). Vectors (1, 2, 0) and (0, 0, 1) span the space and are linearly independent. {(1, 2, 0, (0, 0, 1)} is a basis. Dimension is 2.

(c) (a, 2a, 4a) + (b, 2b, 4b) = (a+b, 2a+2b, 4a+4b) =(a+b, 2(a+b), 4(a+b)). 2nd component is twice 1st, 3rd component four times 1st. Closed under addition. k(a, 2a, 4a) = (ka, 2ka, 4ka). 2nd component is twice 1st, 3rd component four times 1st. Closed under scalar multiplication. Subspace. (a, 2a, 4a) = a(1, 2, 4). {(1, 2, 4)} is a basis. Dimension is 1. Space is a line defined by the vector (1, 2, 4).

(d) (a, -a, 0) + (b, -b, 0) = (a+b, -a-b, 0) = (a+b, -(a+b), 0). 2nd component is negative of 1st. Closed under addition. k(a, -a, 0) = (ka, -ka, 0). 2nd component is negative first. Closed under scalar multiplication. Subspace. (a, -a, 0) = a(1, -1, 0). {(1, -1, 0)} is a basis. Dimension is 1. Space is line defined by the vector (1, -1, 0).

4. (a) (a, b, a) + (c, d, c) = (a+c, b+d, a+c). 3rd component same as 1st. Closed under addition. k(a, b, a) = (ka, kb, ka). 3rd component same as 1st. Closed under scalar multiplication. Subspace. (a, b, a) = a(1, 0, 1) + b(0, 1, 0). {(1, 0, 1), (0, 1, 0)} is a basis. Dimension is 2.

(b) (a, b, 0) + (c, d, 0) = (a+c, b+d, 0). Last component is zero. Closed under addition. k(a, b, 0) = (ka, kb, 0). Last component is zero. Closed under scalar multiplication. Subspace. (a, b, 0) = a(1, 0, 0) + b(0, 1, 0). {(1, 0, 0), (0, 1, 0)} is a basis. Dimension is 2. It is xy-plane.

(c) (a, b, 2) + (c, d, 2) = (a+c, b+d, 4). Last component not 2. Not closed under addition. Not a subspace.

(d) (a, a, a+3) + (b, b, b+3) = (a+b, a+b, a+b+6). Last component is not 1st plus 3. Not closed under addition. Not a subspace.

5. (a) a(1, 2, 3) + b(1, 2, 3) = (a+b)(1, 2, 3). Sum is a scalar multiple of (1, 2, 3). Closed under addition. ka(1, 2, 3) = (ka)(1, 2, 3). It is a scalar multiple of (1, 2, 3). Closed under scalar multiplication. Subspace of $\mathbf{R}^3$. Basis {(1, 2, 3)}. Dimension is 1. Space is line defined by vector (1, 2, 3).

(b) (a, 0, 0) + (b, 0, 0) = (a+b, 0, 0). Last two components zero. Closed under scalar multiplication. k(a, 0, 0) = (ka, o, 0). Last two components zero. Closed under scalar multiplication. Subspace of $\mathbf{R}^3$. Basis {(1, 0, 0)}. Dimension is 1. Space is the x-axis.

(c) (a, 2a) + (b, 2b) = (a+b, 2a+2b) = (a+b, 2(a+b)). 2nd component is twice 1st. Closed under addition. k(a, 2a) = (ka, 2ka). 2nd component is twice 1st. Closed under scalar multiplication. Subspace. Basis {(1, 2)}. Dimension is 1. Space is line defined by vector (1, 2) in $\mathbf{R}^2$.

(d) (a, b, c, 1) + (d, e, f, 1) = (a+d, b+e, c+f, 2). Last component is not 1. Not closed under addition. Not a subspace of $\mathbf{R}^4$.

6. (a) True: Arbitrary vector can be expressed as a linear combination of (1, 0) and (0, 1).
(a, b) = a(1, 0) + b(0, 1).

(b) True: (a, b) = a(1, 0) + b(0, 1) + 0(1, 1). (One or more of the scalars can be zero).

(c) True: p(1, 0) + q(0, 1) = (0, 0) has the unique solution p=0, q=0.

(d) False: Consider the identity p(1, 0) + q(0, 1) + r(0, 2) = (0, 0). Does this have the unique solution p=0, q=0, r=0? No, can have 0(1, 0) + 2(0, 1) - 1(0, 2) = (0, 0). Vectors are not linearly independent - we say that they are linearly dependent.

(e) True: (x, y) = x(1, 0) - y(0, -1). Thus (1, 0) and (0, -1) span $\mathbf{R}^2$. Further, p(1, 0)+q(0, -1)=(0, 0) has the unique solution p=0, q=0. Vectors are linearly independent.

(f) True: (x, y) = $\frac{x}{2}$(2, 0) - $\frac{y}{3}$(0, 3). Thus (2, 0) and (0, 3) span $\mathbf{R}^2$. Further, p(2, 0) + q(0, 3) = (0, 0) ⇔ (2p, 3q) = (0, 0), has the unique solution p=0, q=0. Vectors linearly independent.

7. (a) True: (1, 0, 0) and (0, 1, 0) span the subset of vectors of the form (a, b, 0). Further, p(1, 0, 0) + q(0, 1, 0) = (0, 0, 0) has the unique solution p=0, q=0. Vectors are linearly independent. Subspace is 2D since 2 base vectors. The subspace is the xy plane.

(b) True: The vector (1, 0, 0) spans the subset of vectors of the form a(1, 0, 0). Further p(1, 0, 0)=(0,0,0) has unique solution p=0. Subspace is line defined by vector (1, 0, 0). One vector in basis, thus 1D.

(c) True: Can write (a, 2a, b) in the form (a, 2a, b) = a(1, 2, 0) + b(0, 0, 1).

(d) True: Can write (a, b, 2a-b) in the form (a, b, 2a-b) = a(1, 0, 2) + b(0, 1, -1).

(e) False: 1(1, 0, 0) + 1(0, 1, 0) -1(1, 1, 0) = (0, 0, 0). Thus vectors not linearly independent.

(f) False: $\mathbf{R}^2$ is not a subset of $\mathbf{R}^3$. e.g., (1, 2) is an element of $\mathbf{R}^2$, but not of $\mathbf{R}^3$.

8. (a) Let (x, y) be an arbitrary vector in $\mathbf{R}^2$. Then (x, y) = x(1, 0) + y(0, 1). Thus {(1, 0), (0, 1)} spans $\mathbf{R}^2$. Notice that both vectors are needed - we cannot drop one of them.
(b) (x, y) can be expressed (x, y) = x(1, 0) + y(0, 1) + 0(0, 2), or (x, y) = x(1, 0) + 3y(0, 1) - y(0, 2); there are many ways. Thus {(1, 0), (0, 1), (0, 2)} spans $\mathbf{R}^2$. But it is not an efficient spanning set. The vector (0, 2) is not really needed.
(c) 0(1, 0) + 2(0, 1) - (0, 2) = (0, 0). Thus {(1, 0), (0, 1), (0, 2)} is linearly dependent. It is not a basis.

9. (a) Let (x, y) be an arbitrary vector in $\mathbf{R}^2$. Then (x, y) = x(1, 0) + y(0, 1). Thus {(1, 0), (0, 1)} spans $\mathbf{R}^2$. Notice that both vectors are needed - we cannot drop one of them.
(b) (x, y) can be expressed (x, y) = x(1, 0) + y(0, 1) + 0(1, 1) - or (x, y) = 2x(1, 0) + 2y(0, 1) - y(1,1) - there are many ways. Thus {(1, 0), (0, 1), (1, 1)} spans $\mathbf{R}^2$. But it is not an efficient spanning set. The vector (1, 1) is not really needed.
(c) 1(1, 0) + 1(0, 1) - 1(1, 1) = (0, 0). Thus {(1, 0), (0, 1), (1, 1)} is linearly dependent. It is not a basis.

10. (a) Vectors (1, 0, 0), (0, 1, 0), (0, 0, 1), and (1, 1, 1) span $\mathbf{R}^3$ but are not linearly independent.

(b) Vectors (1, 0, 0), (0, 1, 0) are linearly independent but do not span $\mathbf{R}^3$.

11. Separate the variables in the general solution, (2r - 2s, r - 3s, r, s)= r(2, 1, 1, 0) + s(-2, -3, 0, 1). Vectors (2, 1, 1, 0) and (-2, 3, 0, 1) thus span W.
Also, identity p(2, 1, 1, 0) + q(-2, 3, 0, 1) = (0, 0, 0, 0) leads to p=0, q=0. The two vectors are thus linearly independent. Set {(2, 1, 1, 0), (-2, 3, 0, 1)} is therefore a basis for W.
Dimension 2. Solutions form a plane in $\mathbf{R}^4$.

12. Separate the variables in the general solution, (3r + s, -r - 4s, r, s)= r(3, -1, 1, 0) + s(1, -4, 0, 1). Vectors (3, -1, 1, 0) and (1, -4, 0, 1) thus span W.
Also, identity p(3, -1, 1, 0) + q(1, -4, 0, 1) = (0, 0, 0, 0) leads to p=0, q=0. The two vectors are thus linearly independent. Set {(3, -1, 1, 0), (1, -4, 0, 1)} is therefore a basis for W.
Dimension 2. Solutions form a plane in $\mathbf{R}^4$.

13. (2r, -r, 4r, r) = r(2, -1, 4, r). The set of solutions form a line in $\mathbf{R}^4$. {(2, -1, 4)} is a basis. Dimension is 1.

14. Separate the variables in the solution, (2r - s, -3r - 2s r, s)= r(2, -3, 1, 0) + s(-1, -2, 0, 1). Vectors (2, -3, 1, 0) and (-1, -2, 0, 1) thus span W.
Also, identity p(2, -3, 1, 0) + q(-1, -2, 0, 1) = (0, 0, 0, 0) leads to p=0, q=0. The two vectors are thus linearly independent. The set {(2, -3, 1, 0), (-1, -2, 0, 1)} is therefore a basis for W.
Dimension 2. Solutions form a plane in $\mathbf{R}^4$

<u>Exercise Set 1.5</u>

1. (a) (2,1)·(3,4) = 2x3 + 1x4 = 6 + 4 = 10

   (b) (1,−4)·(3,0) = 1x3 + −4x0 = 3

   (c) (2,0)·(0,−1) = 2x0 + 0x−1= 0

   (d) (5,−2)·(−3,−4) = 5x−3 + −2x−4 = −15 + 8 = −7

2. (a) (1,2,3)·(4,1,0) = 1x4 + 2x1 + 3x0 = 4 + 2 + 0 = 6

   (b) (3,4,−2)·(5,1,−1) = 3x5 + 4x1 + −2x−1 = 15 + 4 + 2 = 21

(c) If (a,b,c) is orthogonal to (5,1,−1), then (a,b,c)·(5,1,−1) = 5a + b − c = 0, so c = 5a+b. Thus any vector of the form (a,b,5a+b) is orthogonal to (5,1,−1).

(d) If (a,b,c,d) is orthogonal to (5,0,1,1), then (a,b,c,d)·(5,0,1,1) = 5a + c + d = 0, so d = −5a−c. Thus any vector of the form (a,b,c,−5a−c) is orthogonal to (5,0,1,1).

(e) If (a,b,c,d) is orthogonal to (6,−1,2,3), then (a,b,c,d)·(6,−1,2,3) = 6a − b + 2c + 3d = 0, so b = 6a+2c+3d. Thus any vector of the form (a,6a+2c+3d, c,d) is orthogonal to (6,−1,2,3).

(f) If (a,b,c,d,e) is orthogonal to (0,−2,3,1,5), then (a,b,c,d,e)·(0,−2,3,1,5) = −2b +3c + d + 5e = 0, so d = 2b−3c−5e. Thus any vector of the form (a,b,c,2b−3c−5e,e) is orthogonal to (0,−2,3,1,5).

20. If (a,b,c) is orthogonal to both (1,2,−1) and (3,1,0), then (a,b,c)·(1,2,−1) = a + 2b − c = 0 and (a,b,c)·(3,1,0) = 3a + b = 0. These equations yield the solution b = −3a and c = −5a, so any vector of the form (a,−3a,−5a) is orthogonal to both (1,2,−1) and (3,1,0).

21. Let (a,b,c) be in W. Then (a,b,c) is orthogonal to (-1,1,1). (a,b,c)·(-1,1,1)=0, -a+b+c=0, c=a-b. W consists of vectors of the form (a,b,a-b). Separate the variables. (a,b,a-b)=a(1,0,1)+b(0,1,-1). (1,0,1), (0,1,-1) span W. Vectors are also linearly independent. {(1,0,1),(0,1,-1)} is a basis for W. The dimension of W is 2. It is a plane spanned by (1,0,1) and (0,1,-1).

22. Let (a,b,c) be in W. Then (a,b,c) is orthogonal to (-3,4,1). (a,b,c)·(-3,4,1)=0, -3a+4b+c=0, c=3a-4b. W consists of vectors of the form (a,b,3a-4b). Separate the variables. (a,b,3a-4b)=a(1,0,3)+b(0,1,-4). {(1,0,3), ((0,1,-4)} is a basis for W. The dimension of W is 2. It is a plane spanned by (1,0,3) and (0,1,-4).

23. Let (a,b,c) be in W. Then (a,b,c) is orthogonal to (1,-2,5). (a,b,c)·(1,-2,5)=0, a-2b+5c=0, a=2b-5c. W consists of vectors of the form (2b-5c,b,c). Separate the variables. (2b-5c,b,c)=b(2,1,0)+c(-5,0,1). {(2,1,0), ((-5,0,1)} is a basis for W. The dimension of W is 2. It is a plane spanned by (2,1,0) and (-5,0,1).

24. Let (a,b,c,d) be in W. Then (a,b,c,d) is orthogonal to (1,-3,7,4).(a,b,c,d)·(1,-3,7,4)=0, a-3b+7c+4d=0, a=3b-7c-4d. W consists of vectors of the form (3b-7c-d,b,c,d). Separate the variables. (3b-7c-4d,b,c,d)=b(3,1,0,0)+c(-7,0,1,0) +d(-4,0,0,1). {(3,1,0,0), (-7,0,1,0), (-4,0,0,1)} is a basis for W. The dimension of W is 3.

25. (a) $d = \sqrt{(6-2)^2+(5-2)^2} = 5.$ (b) $d = \sqrt{(3+4)^2+(1-0)^2} = \sqrt{50} = 5\sqrt{2}.$

(c) $d = \sqrt{(7-2)^2+(-3-2)^2} = \sqrt{50} = 5\sqrt{2}.$ (d) $d = \sqrt{(1-5)^2+(-3-1)^2} = 4\sqrt{2}.$

26. (a) $d = \sqrt{(4-2)^2+(1+3)^2} = \sqrt{20} = 2\sqrt{5}.$ (b) $d = \sqrt{(1-2)^2+(2-1)^2+(3-0)^2} = \sqrt{11}.$

(c) $d = \sqrt{(-3-4)^2+(1+1)^2+(2-1)^2} = \sqrt{54} = 3\sqrt{6}.$

34. $\|u+v\|^2 = (u_1 + v_1)^2 + (u_2 + v_2)^2 + \ldots + (u_n + v_n)^2$

$= u_1^2 + 2u_1 v_1 + v_1^2 + u_2^2 + 2u_2 v_2 + v_2^2 + \ldots + u_n^2 + 2u_n v_n + v_n^2$

$= u_1^2 + u_2^2 + \ldots + u_n^2 + 2u_1 v_1 + 2u_2 v_2 + \ldots + 2u_n v_n + v_1^2 + v_2^2 + \ldots + v_n^2$

$= u_1^2 + u_2^2 + \ldots + u_n^2 + 2(u_1 v_1 + u_2 v_2 + \ldots + u_n v_n) + v_1^2 + v_2^2 + \ldots + v_n^2$

$= \|\mathbf{u}\|^2 + 2(\mathbf{u}\cdot\mathbf{v}) + \|\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$ if and only if $\mathbf{u}\cdot\mathbf{v} = 0$, i.e., if and only if **u** and **v** are orthogonal.

35. $(a,b)\cdot(-b,a) = a \times -b + b \times a = 0$, so $(-b,a)$ is orthogonal to (a,b).

36. $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = (u_1 + v_1)(u_1 - v_1) + (u_2 + v_2)(u_2 - v_2) + \ldots + (u_n + v_n)(u_n - v_n)$

$= u_1^2 - v_1^2 + u_2^2 - v_2^2 + \ldots + u_n^2 - v_n^2$

$= u_1^2 + u_2^2 + \ldots + u_n^2 - v_1^2 - v_2^2 - \ldots - v_n^2 = \|\mathbf{u}\| - \|\mathbf{v}\|$.

Thus $\|\mathbf{u}\| - \|\mathbf{v}\| = 0$ if and only if $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = 0$. That is $\|\mathbf{u}\| = \|\mathbf{v}\|$ if and only if **u** + **v** and **u** - **v** are orthogonal.

37. (a) $\|\mathbf{u}\|^2 = u_1^2 + u_2^2 + \ldots + u_n^2 \geq 0$, so $\|\mathbf{u}\| \geq 0$.

(b) $\|\mathbf{u}\| = 0$ if and only if $u_1^2 + u_2^2 + \ldots + u_n^2 = 0$ if and only if $u_1 = u_2 = \ldots = u_n = 0$.

(c) $\|c\mathbf{u}\|^2 = (cu_1)^2 + (cu_2)^2 + \ldots + (cu_n)^2 = c^2(u_1^2 + u_2^2 + \ldots + u_n^2) = c^2 \|\mathbf{u}\|^2$, so $\|c\mathbf{u}\| = |c|\,\|\mathbf{u}\|$.

38. (a) $\|\mathbf{u}\| = |u_1| + |u_2| + \ldots + |u_n| \geq 0$ since each term is equal to or greater than zero.

$|u_1| + |u_2| + \ldots + |u_n| = 0$ if and only if each term is zero.

$\|c\mathbf{u}\| = |cu_1| + |cu_2| + \ldots + |cu_n| = |c||u_1| + |c||u_2| + \ldots + |c||u_n|$

$= |c|(|u_1| + |u_2| + \ldots + |u_n|) = |c|\,\|\mathbf{u}\|$.

$\|(1,2)\| = |1| + |2| = 3$, $\|(-3,4)\| = |-3| + |4| = 7$, $\|(1,2,-5)\| = |1| + |2| + |-5| = 8$,

and $\|(0,-2,7)\| = |0| + |-2| + |7| = 9$.

7.
$$\begin{aligned} I_1 + I_2 - I_3 &= 0 \\ 2I_1 + 4I_3 &= 34 \\ 4I_2 + 4I_3 &= 28 \end{aligned}$$
so that $I_1 = 5$, $I_2 = 1$, $I_3 = 6$.

8.
$$\begin{aligned} I_1 + I_2 - I_3 &= 0 \\ I_1 + 2I_3 &= 9 \\ 3I_2 + 2I_3 &= 17 \end{aligned}$$
so that $I_1 = 1$, $I_2 = 3$, $I_3 = 4$.

9.
$$\begin{aligned} I_1 + I_2 - I_3 &= 0 \\ 3I_3 &= 9 \\ 4I_2 + 3I_3 &= 13 \end{aligned}$$
so that $I_1 = 2$, $I_2 = 1$, $I_3 = 3$.

10.
$$\begin{aligned} I_1 + I_2 - I_3 &= 0 \\ 2I_1 + 2I_3 &= 4 \\ 4I_2 + 2I_3 &= 2 \end{aligned}$$
so that $I_1 = 1$, $I_2 = 0$, $I_3 = 1$.

11.
$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ I_1 + 3I_2 &= 31 \\ I_1 + 7I_3 &= 31 \end{aligned}$$
so that $I_1 = 10$, $I_2 = 7$, $I_3 = 3$.

12.
$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ I_3 - I_4 - I_5 &= 0 \\ I_2 &= 4 \\ I_4 &= 4 \\ I_5 &= 4 \end{aligned}$$
so that $I_1 = 12$, $I_2 = 4$,
$I_3 = 8$, $I_4 = 4$, $I_5 = 4$.

13.
$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ I_3 - I_4 + I_5 &= 0 \\ I_1 + I_2 &= 4 \\ I_1 + 2I_4 &= 4 \\ 2I_4 + 2I_5 &= 2 \end{aligned}$$
so that $I_1 = 7/3$, $I_2 = 5/3$,
$I_3 = 2/3$, $I_4 = 5/6$, $I_5 = 1/6$.

14. Assume I_3 flows from A to B.
$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ I_1 + 2I_3 &= 4 \\ I_2 - 2I_3 &= 9 \end{aligned}$$
gives $I_1 = 6$, $I_2 = 7$, $I_3 = -1$, so the current in AB is 1 amp flowing from B to A.

15. Let I_1 be the current in the direction from the 16volt battery to A, let I_2 be the current from A to B, and let I_3 be the current in the direction from C to B.
$$\begin{aligned} I_1 - I_2 - I_3 &= 0 : 0 \\ 5I_1 + I_2 &= 16 : 16 \\ -I_2 + 5I_3 &= 9 : 23 \end{aligned}$$

17. A: $x_1 - x_4 = 100$ B: $x_1 - x_2 = 200$

C: $-x_2 + x_3 = 150$ D: $x_3 - x_4 = 50$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 100 \\ 1 & -1 & 0 & 0 & 200 \\ 0 & -1 & 1 & 0 & 150 \\ 0 & 0 & 1 & -1 & 50 \end{bmatrix} \approx \dots \approx \begin{bmatrix} 1 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & -1 & -100 \\ 0 & 0 & 1 & -1 & 50 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = x_4 + 100$, $x_2 = x_4 - 100$, $x_3 = x_4 + 50$.

$x_2 = 0$ is min flow along BC; i.e., BC, closed to traffic.
In that case $x_4 = 100$, $x_1 = 200$, $x_3 = 150$. Then alternative route would have to be provided to get from B to towns in direction of C, and D.

18. $x_1 = x_2+100$, $x_3=x_2+90$, $x_3=x_4+130$, $x_5=x_4+110$, $x_5=x_6+80$, $x_7=x_6+75$, $x_7=x_8+120$, $x_1=x_8+155$. Since a flow cannot be negative these equs give:
$x_1 \geq 100$, $x_3 \geq 90$, $x_3 \geq 130$, $x_5 \geq 110$, $x_5 \geq 80$, $x_7 \geq 75$, $x_7 \geq 120$, $x_1 \geq 155$.
Thus, must have $x_1 \geq 155$, $x_3 \geq 130$, $x_5 \geq 110$, $x_7 \geq 120$.
Is $x_1=155$ possible, i.e., does it result in nonneg flows? Yes, gives
$x_2=55$, $x_3=145$, $x_4=15$, $x_5=125$, $x_6=45$, $x_7=120$, $x_8=0$.
Minimum flow allowable along x_1 is 155. Note that this is attained by closing x_8 to traffic. Alternative routes (clearly labelled diversions!) will then have to be provided for the x_7 traffic wanting to get to some of the towns that are accessed from other exits of the roundabout.

19. A: $x_1 + x_2 = 200$ B: $x_1 - x_3 - x_4 = 0$

E: $x_2 + x_3 - x_5 = 0$ D: $x_4 + x_5 = 200$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 200 \\ 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 200 \end{bmatrix} \approx \dots \approx \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 200 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$x_1 = x_3 - x_5 + 200$, $x_2 = -x_3 + x_5$, $x_4 = -x_5 + 200$.

Total time $= k(x_1 + 2x_2 + x_3 + 2x_4 + x_5)$
$= 4(x_3 - x_5 + 200 + 2(-x_3 + x_5) + x_3 + 2(-x_5 + 200) + x_5) = 4(600) = 2400$ minutes.

This averages to 12 minutes per car. It is interesting to note that the time is independent of the actual distribution of traffic for this model. This will not be true if k differs on different stretches of road due to different road conditions. Students can examine these situations.