Homework 13

Section 5.1

7. $(3,-1,7) = a(2,0,-1) + b(0,1,3) + c(1,1,1)$, so $3 = 2a+c$, $-1 = b+c$, and $7 = -a+3b+c$. This system of equations has the unique solution $a = 16/3$, $b = 20/3$, and $c = -23/3$, so

$$\mathbf{u}_B = \begin{bmatrix} 16/3 \\ 20/3 \\ -23/3 \end{bmatrix}.$$

17. $P = \begin{bmatrix} 4 & 3 \\ 1 & -1 \end{bmatrix}$, and $\mathbf{u}_{B'} = P\mathbf{u}_B = \begin{bmatrix} 12 \\ 3 \end{bmatrix}$, $\mathbf{v}_{B'} = P\mathbf{v}_B = \begin{bmatrix} 11 \\ 1 \end{bmatrix}$, and $\mathbf{w}_{B'} = P\mathbf{w}_B = \begin{bmatrix} -5 \\ -3 \end{bmatrix}$.

22. The transition matrix from B' to B is $P = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, so the transition matrix from B to B' is

$P^{-1} = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$, and $\mathbf{u}_{B'} = P^{-1}\mathbf{u}_B = \begin{bmatrix} -5 \\ -13 \end{bmatrix}$.

24. The transition matrix from B to the standard basis is $R = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$ and the transition matrix from B' to the standard basis is $Q = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}$. $P = Q^{-1}R = -\begin{bmatrix} 2 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -7 & -1 \\ 17 & 3 \end{bmatrix}$, and $\mathbf{u}_{B'} = P\,\mathbf{u}_B = \begin{bmatrix} -47 \\ 113 \end{bmatrix}$.

Section 5.2

3. $T(2,1) = T(2(1,0) + (0,1)) = 2T(1,0) + T(0,1) = 2(2,5) + (1,-3) = (5,7)$. Alternatively, the matrix of T with respect to the standard basis is $A = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix}$. $A\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$, so $T(2,1) = (5,7)$.

12. $T(\mathbf{u}_1) = (1,4,6) = 1\mathbf{u}_1' + 2\mathbf{u}_2' - 6\mathbf{u}_3'$ and $T(\mathbf{u}_2) = (4,3,-2) = 4\mathbf{u}_1' + 3/2\mathbf{u}_2' + 2\mathbf{u}_3'$, so

$A = \begin{bmatrix} 1 & 4 \\ 2 & 3/2 \\ -6 & 2 \end{bmatrix}$. $\mathbf{u} = (9,1) = 1\mathbf{u}_1 + 2\mathbf{u}_2$, so the coordinate vector of $\mathbf{u}$ relative to the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and the coordinate vector of $T(\mathbf{u})$ relative to the basis $\{\mathbf{u}_1', \mathbf{u}_2', \mathbf{u}_3'\}$ is

$A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ -2 \end{bmatrix}$. Thus $T(\mathbf{u}) = 9\mathbf{u}_1' + 5\mathbf{u}_2' - 2\mathbf{u}_3' = (9,10,2)$.