Name: ______________________ ☐ TTH ☐ MWF 10AM ☐ MWF 11AM

Pledge: ____

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Signature: ______________________________

To get credit for a problem, you must show all of your reasoning and calculations. No calculators may be used.

Problem	Score
Multiple Choice	
1	
2	
3	
4	
5	
6	
Total	

Multiple Choice Answers

1	
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Multiple Choice (5 Points Each)

1. Find the dot product of $\mathbf{u} = [1, 3, 7]$ and $\mathbf{v} = [-6, 2, 1]$.

   (a) 7 (b) 0 (c) -7 (d) 1 (e) 4

2. If A is an invertible $n \times n$ matrix, then which of the following are true?

   i The columns are linearly independent.

   ii The equation $A\mathbf{x} = \mathbf{b}$ might not have a solution for some $\mathbf{b}$.

   iii The rank of A is n.

   (a) i only (b) ii & iii only (c) i & ii only (d) i & iii only (e) all are true.

3. Let R be the matrix

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

   The system $R\mathbf{x} = \mathbf{0}$ has

   (a) 1 solution (b) 0 solutions (c) 2 solutions (d) infinitely many solutions (e) not enough information.

4. If A is a 2×4 matrix of rank 2, then the dimension of the kernel of $L(\mathbf{v}) = A\mathbf{v}$ is

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 3 & 2 \end{bmatrix},$$

   then $B - A =$

   (a) $\begin{bmatrix} 4 & 2 & 0 \\ 2 & 0 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & -2 & 0 \\ -2 & 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

6. If A is a 3×3 matrix with $|A| = 4$, and if B is the result of scaling the first row of A by 3, then $|B| =$

   (a) 4 (b) 12 (c) -12 (d) 0 (e) Not enough information.

7. The eigenvalues of $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ are

   (a) 3, 4 (b) 0, 7 (c) ± 3 (d) ± 4 (e) ± 5

8. Find all values of k such that the set

$$\{[1,1,1],[2,k,4],[3,6,k]\}$$

does *not* form a basis for $\mathbb{R}^3$. $k =$

(a) 5 (b) -5 (c) 0, 5 (d) 2 (e) It's always a basis.

For the next two problems, let

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 2 & 4 & 3 & 3 \\ 3 & 6 & 4 & 4 \end{bmatrix}.$$

9. A basis for the kernel of the map $L(\mathbf{v}) = A\mathbf{v}$ is given by

(a) $\{[-2,1,0,0],[0,0,-1,1]\}$ (b) $\{[-2r,r,s,-s]\}$ (c) $\{[1,2,0,0],[0,0,1,1]\}$
(d) $\{[1,1,2,-1],[1,2,3,2],[2,2,4,2]\}$ (e) $\{[0,0,1,-1]\}$

10. A basis for the column space of A is given by

(a) $\{[-1,-1,1,0],[-2,-1,0,1]\}$ (b) $\{[-r-2s,-r-s,r,s]\}$ (c) $\{[1,0,1,2],[0,1,1,1]\}$
(d) $\{[1,1,2,-1],[1,2,3,2],[2,2,4,2]\}$ (e) $\{[0,0,1,-1]\}$

11. Let $f(x) = x^2$. Then the second order Fourier approximation to f over the interval $[-\pi,\pi]$ is

(a) $2\sin x - \sin 2x$ (b) $\frac{\pi^2}{3} - 4\cos x + \cos 2x$ (c) $1 + \cos x + \cos 2x$
(d) $\frac{4\pi^3}{3} - 4\pi\cos x + \pi\cos 2x$ (e) Cannot be determined.

12. If $p(x) = x^2 + 3x + 5 \in P_2(x)$, and if $\mathcal{B} = \{x+1, 2, x^2+x+1\}$, then $[p]_{\mathcal{B}} =$

(a) $[1,3,5]$ (b) $[2,2,1]$ (c) $[2,1,2]$ (d) $[2,1,1]$ (e) $[1,2,1]$

13. Which of the following are subspaces of $P_3(x)$?

i $\{p(x) | p''(x) + p'(x) + p(x) = 0\}$

ii $\{p(x) | p(3) = 3\}$

iii $\{p(x) | p(4) = 0\}$

iv $\{p(x) | p'(x) = p(0)\}$

(a) i only (b) iii only (c) i & iv only (d) i, ii, & iii only (e) i, iii, & iv only.

14. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$, then the adjoint of A is $adj(A) =$

(a) $\begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$

15. If $\mathbf{v}_1 = x^3 + x^2 + x$, $\mathbf{v}_2 = x^2 + x + 1$, and $\mathbf{v}_3 = x^3 + x + 1$, then express $3x^2 + 2x + 1$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, & $\mathbf{v}_3$:

(a) $\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$ (b) $\mathbf{v}_1 + 2\mathbf{v}_2 - \mathbf{v}_3$ (c) $\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3$ (d) $-\mathbf{v}_1 + 2\mathbf{v}_2 + \mathbf{v}_3$ (e) Not in the span.

16. Let $Q(x, y) = 4x^2 + 6xy - 4y^2$. Then the graph of $Q(x, y) = 1$ is an

(a) ellipse (b) parabola (c) hyperbola (d) line (e) None of these.

1. (a) (12 Points)Let

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & -2 & 3 \\ 4 & 8 & a \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 5 \\ 7 \\ c \end{bmatrix}.$$

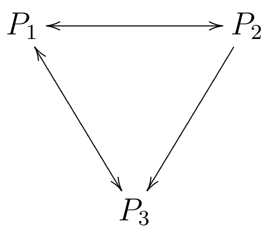
For what values of a and c does the system $A\mathbf{x} = \mathbf{b}$ have

(i) Many solutions?

(ii) A unique solution?

(iii) No solutions?

(b) (8 Points) Find the adjacency matrix A of the following communication network. Find the number of 2-paths from P_1 to P_2, from P_1 to P_3, and from P_2 to P_3 using the matrix A^2



2. (a) (6 Points) Find an LU decomposition (if possible) of the matrix

$$C = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ -1 & 5 & 9 \end{bmatrix}$$

(b) (10 Points) Solve the system $A\mathbf{x} = \mathbf{b}$ using the LU factorization $A = LU$;

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 4 & -1 & 9 \end{bmatrix}, L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$$

(c) (4 Points) Using Cramer's rule solve the following system

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ -x_1 + x_2 &= 5 \end{aligned}$$

3. (a) (10 Points) Find an orthogonal matrix C such that

$$C^{-1}AC = \text{a diagonal matrix, for } A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}.$$

(b) (10 Points) Find the characteristic polynomial and the eigenvalues of

$$B = \begin{bmatrix} 3 & 0 & 0 & 13 \\ -25 & 7 & 11 & -6 \\ 18 & 0 & 1 & 5 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$

4. (a) (10 Points) Find the least square solution to the following system

$$\begin{aligned} x_1 + x_2 &= 4 \\ -x_1 + x_2 &= 0 \\ -x_2 &= 1 \\ x_1 &= 2 \end{aligned}$$

(b) (10 Points) Let $<,>$ be the inner product on P_2 defined by

$$\langle P, Q \rangle = \int_0^1 P(x)Q(x)dx.$$

Let $P(x) = x - 1,\ Q(x) = x + 1$ and angle between $P(x)$ and $Q(x)$ be θ. Find $\cos\theta$.

5. (a) (10 Points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x + y, x - 2y)$. Find the inverse of T.

   (b) (10 Points) Let $S\colon \mathbb{R}^2 \to \mathbb{R}^3$ be defined by

$$S(x, y) = (-6x + 4y, -6x + 3y, -3x + 2y).$$

   Let

$$\mathcal{B} = \{(1, 2), (2, 3)\},$$
$$\mathcal{B}' = \{(2, 0, 0), (0, -3, 0), (0, 0, 1)\}$$

   be bases for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively.

   Find the matrix representation A of S with respect to $\mathcal{B}$ and $\mathcal{B}'$.

6. Consider the bases

$$\mathcal{B} = \{2, x+1, x^2\}$$
$$\mathcal{B}' = \{1, x, x+x^2\}$$

for P_2. Let $p = 5x^2 + 3x + 2$.

(a) (10 Points) Find $p_{\mathcal{B}}$ and $p_{\mathcal{B}'}$.

(b) (10 Points) Find the transition matrix P from $\mathcal{B}$ to $\mathcal{B}'$. Compute $p_{\mathcal{B}'}$ using P and $p_{\mathcal{B}}$.