

Basis Practice *Solutions*

Find a basis for each of the following homogeneous linear systems.

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ 2 & 4 & 1 & 10 & 0 \\ 3 & 6 & 2 & 17 & 0 \\ 4 & 8 & -2 & -8 & 0 \end{array} \right) \xrightarrow{-2} \xrightarrow{-4} \quad (1) \Rightarrow$$

$$\xrightarrow{-2} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 2 & 8 & 0 \\ 0 & 0 & -2 & -20 & 0 \end{array} \right) \xrightarrow{-2} \Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -12 & 0 \end{array} \right) \xrightarrow{-12 \div} \Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{-4} \xrightarrow{-3} \Rightarrow$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow \begin{array}{l} x, z, w \text{ lead,} \\ y \text{ free} \Rightarrow y = s \end{array} \quad \text{Sol:} \quad \begin{bmatrix} -2s \\ s \\ 0 \\ 0 \end{bmatrix} = s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \text{basis} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$\xrightarrow{-1} \left(\begin{array}{ccccc|c} 0 & 1 & 2 & 2 & 11 & 0 \\ 0 & 2 & 5 & 7 & 33 & 0 \\ 0 & 1 & 2 & 3 & 14 & 0 \end{array} \right) \xrightarrow{-2} \quad (2)$$

$$\xrightarrow{-2} \left(\begin{array}{ccccc|c} 0 & 1 & 2 & 2 & 11 & 0 \\ 0 & 0 & 1 & 3 & 11 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{array} \right) \xrightarrow{-3} \Rightarrow \left(\begin{array}{ccccc|c} 0 & 1 & 2 & 0 & 5 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{array} \right) \xrightarrow{-2} \Rightarrow \left(\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{array} \right)$$

$$\begin{array}{l} y, z, w \text{ lead} \Rightarrow \\ x, u \text{ free} \quad x = s, u = t: \end{array} \quad \text{Sol:} \quad \begin{bmatrix} s \\ -t \\ -2t \\ -3t \\ t \end{bmatrix} = s \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ -2 \\ -3 \\ 1 \end{bmatrix} \Rightarrow \text{basis} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -2 \\ -3 \\ 1 \end{bmatrix} \right\}$$

$$\xrightarrow{4} \left(\begin{array}{ccc|c} -2 & -2 & -1 & 0 \\ 3 & 3 & 1 & 0 \\ 8 & 8 & 7 & 0 \end{array} \right) \xrightarrow{3/2} \quad (3)$$

$$\left(\begin{array}{ccc|c} -2 & -2 & -1 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 3 & 0 \end{array} \right) \xrightarrow{\div -2} \xrightarrow{\div -1/2} \xrightarrow{\div 3} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1/2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) \xrightarrow{-1} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) \Rightarrow \begin{array}{l} x, z \text{ lead,} \\ y \text{ free} \Rightarrow y = s \end{array}$$

$$\text{Sol} = \begin{bmatrix} -s \\ s \\ 0 \end{bmatrix} = s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \text{basis} = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$