Goal: understand equivariant commutative ring spectra. Have an equivariant spectrum $\mathcal{R}$ + a homotopy coherent comm. multiplication + homotopy coherent norms.

What does structure do we have and what does this mean for $G$-categories in general?

Step back: what is this norm thing?

In genuine $G$-spectra, $\mathcal{D}_G$ objects = spectra of $G$-colours, morphisms: $G$-spaces/spectra of maps of ring algebras. 

Slightly different from the usual definition $S^2$, $S^{2n} \mapsto S^n$, for some $V$. Lewis showed this has the same information (he did so with the transfer).

$\mathcal{D}_G$ is a closed symmetric monoidal category \( \mathcal{D}_G \otimes \mathcal{D}_G \to \mathcal{D}_G \), symmetric monoidal in both factors.

In particular, these are torsor over $G$-sets/spaces: $G$-Ens $(X \otimes Y, R) = \text{Top}(X, \text{Comm}(T, R))$.

The (H-hip-hooray) The tensoring map extends to $- \otimes : \text{Ens}(G \otimes H, \mathcal{D}_G) \to \mathcal{D}_G$, symmetric monoidal in both factors.

For $G/H$, $G/H \otimes X, \mathcal{D}_G(x, y) \to \mathcal{D}_G$ is the norm used often in the known proof:

1. $\mathcal{D}_G$ is symmetric monoidal
2. $\mathcal{D}_G(x, y) \to \mathcal{D}_G$ is the norm used often in the known proof.

$\mathcal{D}_G$ is symmetric monoidal (we can speak of a $G$-set, not just $G$-set).

This activity adds something valuable, so we can talk about commutative rings again:

Let $\mathcal{F}$ be a category of subgroups (He $\equiv \mathcal{K} \equiv G$, $G \equiv G^\bullet \text{algebras}$).

This has a cat $\mathcal{D}_G$: obj is $\{X \mid \text{abcr}(X) \in \mathcal{F} \forall X \in X\}$.

Def: If $C$ is $G$-symmetric monoidal, the $\mathcal{F}$ is an $F$-comm monoidal if $\mathcal{D}_G$ extends to a functor $\mathcal{D}_G \to C$.

Def: If $C$ is $\mathcal{D}_G$, this is always genuinely commutative! We want a homotopical version, and I will come back to this.

Question: Why does Bousfield localise preserve $(\mathcal{F})$-commutative?

(H-Hip-hooray) Why is the category of $G$-sets instead of $G$-sets?

This is a different version of the result than the discussed before, but the difference is mainly expository.

In a $G$-spectra, $ZG$-algebras, $\mathcal{D}_G$.

The dual map: $\wedge \otimes \mathcal{D}_G \otimes \wedge \mathcal{D}_G$ commutes.

Let $U$ be a $G$-inverse, then $(U \otimes U) \to (U \otimes U)$ is a closed in $G$-spacs, weakened by an $E_0$-closed $\mathcal{D}_G$-algebras over this and the homotopical versions have no preferred product or norm, etc.
Model tells us that if $R$ is an $E_1$-ring spectrum, then the category of modules is symmetric monoidal. So a "dual" to the category of objects gives us the structure of the set of modules: If $R$ is a commutative $F$-algebra, then the category of modules has $-\otimes_F$: $\text{Mod}_F \times \text{R-Mod} \to \text{R-Mod}$. If $R$ is commutative $G$-spectrum, then $\text{R-Mod}$ has norms:

- $N^G_M$ is naturally a $N^G_{R \otimes \text{H} N} \cdot \text{R-Mod}$.
- $R_{\otimes F}$ on $\text{R-Mod}$.

Algebra: Two candidates: $G$-Mod, $G^\text{ Mackey}$.
- Both are (known) equivariant cats, auto-cartesian, $G$-symmetric monoidal:
- $N^G_{R \otimes \text{H} N} \cdot \text{Mod}$.
- (Mackey dual)

$G$-Mod has a similar dual back to spectra; a comm ring obj is automatically $G$-comm. This is just too weak.

$G^\text{ Mackey}$ has more flexibility. In particular, we have examples of $F$-algebras:

- $\mathcal{I}^F \cong \mathcal{C}_G$ (the dual to the augmentation ideal). This is $\mathcal{I}_{\alpha} S^\lambda_+ \mathcal{S}^\lambda_+$ for $\mathcal{I}$-acting rep.

- $\mathcal{I}^F$ is a comm ring ($S^\lambda_+$ is homotopy comm).

- $N^{\mathcal{C}_G}_{\mathcal{C}_G} \mathcal{I}^F \mathcal{C}_G = \mathcal{I}^F (\mathcal{I}_{\alpha} \mathcal{S}^\lambda_+ \mathcal{S}^\lambda_+ + \text{Hom}(\mathcal{I}_{\alpha} \mathcal{S}^\lambda_+, \mathcal{S}^\lambda_+))$. $\mathcal{I}^F$ is a $G$-algebra, $\mathcal{I} \cong \mathcal{C}_G$.

- $N^{\mathcal{C}_G}_{\mathcal{C}_G} \mathcal{C}_G = 0$.

The usual computation produces a family of examples, one for any cofinite in $G$.

This is a special case: $F = \mathbb{N}$. $F$-algebras have another name in the literature: Tambara functors. So we get extra structure on the category of modules over a Tambara functor, namely norms.

Still missing something. $\otimes$, the monoidal product in $G^\text{Mackey}$ is the Kan extension of $\otimes$ in $\text{Mod}$ over $\otimes$ in $\text{Gr}$. Both $\text{Mod}$ and $\text{Gr}$ are $G$-symmetric monoidal. So we should just repeat Kan extended in the "right" $G$-way, but I don't yet know how.