Worksheet 04 Solutions

Instructor: Hong Chen **Topics:** Sections 3.2–3.3 and 3.7

1. Power and linearity rules

(a) $f(x) = 3\sqrt{x} + \dfrac{2}{x} - \dfrac{1}{x^2}$, $x > 0$

Rewrite the function as $f(x) = 3x^{1/2} + 2x^{-1} - x^{-2}$. Differentiate each term:

$$f'(x) = \frac{3}{2}x^{-1/2} - 2x^{-2} + 2x^{-3} = \boxed{\frac{3}{2\sqrt{x}} - \frac{2}{x^2} + \frac{2}{x^3}}, \qquad x > 0.$$

(b) $g(x) = e^2(x^2 - 3x) + 2e^x$

Since e^2 is a constant, the constant multiple and sum rules give

$$\boxed{g'(x) = e^2(2x - 3) + 2e^x}.$$

The derivative of e^2 is 0, while the derivative of e^x is e^x.

2. Products, quotients, and a tangent line

(a) $p(x) = (x^2 - 3x + 1)e^x$

Apply the Product Rule:

$$\begin{aligned} p'(x) &= (2x - 3)e^x + (x^2 - 3x + 1)e^x \\ &= \boxed{e^x(x^2 - x - 2)}. \end{aligned}$$

(b) $q(x) = \dfrac{x^2 + 1}{x + 2}$, $x \neq -2$

Apply the Quotient Rule:

$$q'(x) = \frac{(x + 2)(2x) - (x^2 + 1)(1)}{(x + 2)^2} = \boxed{\frac{x^2 + 4x - 1}{(x + 2)^2}}, \qquad x \neq -2.$$

(c) Tangent line to $y = q(x)$ at $x = 0$

The point and slope are

$$q(0) = \frac{0^2 + 1}{0 + 2} = \frac{1}{2}, \qquad q'(0) = \frac{0^2 + 4(0) - 1}{(0 + 2)^2} = -\frac{1}{4}.$$

Thus the tangent line is $y - 1/2 = -(1/4)(x - 0)$, or

$$\boxed{y = \frac{1}{2} - \frac{1}{4}x}.$$

3. Compositions and the chain rule

(a) $y = (2t^3 - t + 1)^4$

The inner function is $u = 2t^3 - t + 1$, and the outer function is $y = u^4$. Their derivatives are

$$\frac{du}{dt} = 6t^2 - 1, \qquad \frac{dy}{du} = 4u^3.$$

By the Chain Rule,

$$\frac{dy}{dt} = \frac{dy}{du}\bigg|_{u=2t^3-t+1} \frac{du}{dt} = \boxed{4(2t^3 - t + 1)^3(6t^2 - 1)}.$$

(b) $y = \dfrac{1}{\sqrt{x^2 + 4}}$

Write $y = (x^2 + 4)^{-1/2}$. The inner function is $u = x^2 + 4$, and the outer function is $y = u^{-1/2}$. Since $u > 0$ for every real x, the Chain Rule applies everywhere:

$$\frac{dy}{dx} = -\frac{1}{2}(x^2 + 4)^{-3/2}(2x) = \boxed{-\frac{x}{(x^2 + 4)^{3/2}}}.$$

(c) $y = e^{x^2 - 2x}$

The inner function is $u = x^2 - 2x$, and the outer function is $y = e^u$. Using $du/dx = 2x - 2$ and $dy/du = e^u$,

$$\frac{dy}{dx} = e^{x^2-2x}(2x - 2) = \boxed{(2x - 2)e^{x^2-2x}}.$$

(d) $y = \left(\dfrac{x + 2}{x - 2}\right)^5$, $x \neq 2$

The inner function is $u = (x + 2)/(x - 2)$, and the outer function is $y = u^5$. First use the Quotient Rule on the inner function:

$$\frac{du}{dx} = \frac{(x - 2)(1) - (x + 2)(1)}{(x - 2)^2} = -\frac{4}{(x - 2)^2}.$$

Then apply the Chain Rule:

$$\frac{dy}{dx} = 5\left(\frac{x + 2}{x - 2}\right)^4 \left(-\frac{4}{(x - 2)^2}\right)$$

$$= \boxed{-\frac{20(x + 2)^4}{(x - 2)^6}}, \qquad x \neq 2.$$

4. Derivatives from a table

Use the given values:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	2	1	-1	3
1	-1	2	4	-2
2	3	-1	5	1

(a) $(fg)'(1)$

The Product Rule gives

$$(fg)'(1) = f'(1)g(1) + f(1)g'(1) = (4)(2) + (-1)(-2) = \boxed{10}.$$

(b) $(f/g)'(1)$

Since $g(1) = 2 \neq 0$, the Quotient Rule gives

$$(f/g)'(1) = \frac{g(1)f'(1) - f(1)g'(1)}{g(1)^2} = \frac{2(4) - (-1)(-2)}{2^2} = \boxed{\frac{3}{2}}.$$

(c) $h'(0)$**, where** $h(x) = f(g(x))$

The inner output is $g(0) = 1$, so use $f'(1) = 4$. The Chain Rule gives

$$h'(0) = f'(g(0))g'(0) = f'(1)g'(0) = (4)(3) = \boxed{12}.$$

(d) $k'(0)$**, where** $k(x) = g(f(x))$

Now the inner output is $f(0) = 2$, so use $g'(2) = 1$. Thus

$$k'(0) = g'(f(0))f'(0) = g'(2)f'(0) = (1)(-1) = \boxed{-1}.$$