Worksheet 03

Instructor: Hong Chen **Topics:** Sections 2.6–2.8 and 3.1

Due: Sunday, October 18, at 11:59 p.m. **Submit:** Gradescope

Show your work and explain your reasoning. Worksheets are assessed for completion.

Limits and end behavior

1. Evaluate the limits. In (a), give explicit lower and upper bounds for the Squeeze Theorem. In (b)–(d), use the important trigonometric limits.

(a) $\lim_{x\to 0} x \sin \dfrac{1}{x^2}$

(b) $\lim_{x\to 0} \dfrac{\sin 5x}{\tan 2x}$

(c) $\lim_{x\to 0} \dfrac{1-\cos 4x}{x}$

(d) $\lim_{x\to 0} \dfrac{\sin 3x \sin 2x}{x^2}$

2. For each function, find its limits as $x \to \infty$ and $x \to -\infty$, and state all horizontal asymptotes.

(a) $R(x) = \dfrac{2x^2 - 3x + 1}{x^2 + 4}$

(b) $S(x) = \dfrac{3x}{\sqrt{x^2 + 4}}$

In (b), explain why factoring x^2 out of the square root introduces $|x|$.

The Intermediate Value Theorem

3. Let $p(x) = x^3 + x - 3$.

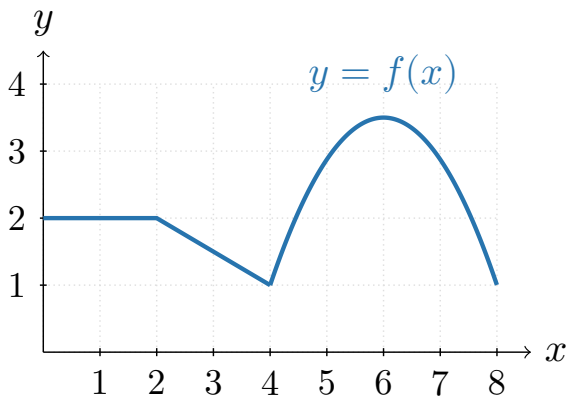
(a) Use the IVT to show that $p(x) = 0$ has a solution in $(1, 2)$. State the continuity hypothesis and check the endpoint values.

(b) Does the IVT by itself show that there is exactly one solution? Explain.

(c) A student claims that $q(x) = 1/x$ has a zero in $[-1, 1]$ because $q(-1) < 0 < q(1)$. Explain the error.

Derivatives from a graph

4. Use the graph of f to answer the questions. Read $f'(a)$ as the slope of the tangent line at $x = a$.



(a) Find $f'(1)$, $f'(3)$, and $f'(6)$.

(b) On the interior of the displayed domain, where is $f'(x) < 0$, and where is $f'(x) > 0$?

(c) Which is larger, $f'(4.5)$ or $f'(5.5)$? Explain from the graph.

(d) Is f differentiable at $x = 2$? Compare the slopes on the two sides.