

Worksheet 02

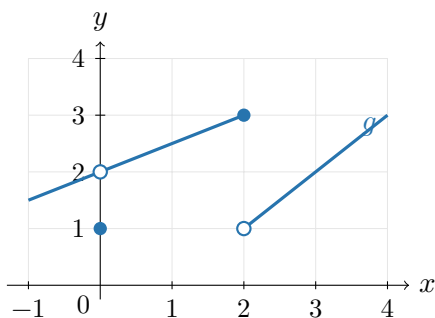
Instructor: Hong Chen

Topics: Sections 2.2–2.5

Due: Sunday, October 11, at 11:59 p.m. **Submit:** Gradescope

Limits and continuity

1. The graph of g is shown below. Open circles are excluded points; filled dots show function values.



- (a) Complete the table. Write DNE if a limit does not exist.

c	$g(c)$	$\lim_{x \rightarrow c^-} g(x)$	$\lim_{x \rightarrow c^+} g(x)$	$\lim_{x \rightarrow c} g(x)$
0				
2				

- (b) At each of $c = 0$ and $c = 2$, classify the discontinuity and state whether g is left-continuous, right-continuous, or neither. Which discontinuity can be removed by changing only $g(c)$? Give the new value.

2. Let c be a constant and define

$$f(x) = \begin{cases} x^3 + c, & x \leq 2, \\ \frac{8}{x} - 3c, & x > 2. \end{cases}$$

- (a) Where is f definitely continuous, regardless of c ? Explain why the term $8/x$ does not cause a problem at $x = 0$ for this piecewise function.
- (b) Find $\lim_{x \rightarrow 2^-} f(x)$, $\lim_{x \rightarrow 2^+} f(x)$, and $f(2)$ in terms of c .
- (c) Find c so that f is continuous on $\mathbb{R}$. Verify continuity at 2.

Indeterminate forms

3. Identify the following indeterminate forms and evaluate the limits:

(a) $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^2 - 9}$

(b) $\lim_{x \rightarrow 0^+} \frac{\sqrt{4 + 3x} - 2}{x}$

(c) $\lim_{x \rightarrow 1^+} \frac{\frac{3}{(x-1)^2} + \frac{2}{x-1}}{\frac{2}{(x-1)^2} - 1}$

(d) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x^2} - \frac{1}{x^2 + x^4} \right)$

(e) $\lim_{x \rightarrow 0^+} x \left(\frac{1}{x} + 2 \right)$

(f) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 + x - 6}$