

Math 31A – Worksheet 1 Solutions

Algebra Review Fall 2026

1. Solve the equations in $\mathbb{R}$

For $ax^2 + bx + c = 0$ with $a \neq 0$, the quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Real solutions exist exactly when the discriminant $b^2 - 4ac$ is nonnegative.

(a) For $x^2 + x - 6 = 0$, take $a = 1$, $b = 1$, and $c = -6$:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)} = \frac{-1 \pm 5}{2}.$$

Thus $x = -3$ or $x = 2$.

(b) Rewrite the equation as $2x^2 + x - 10 = 0$, so $a = 2$, $b = 1$, and $c = -10$:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-10)}}{2(2)} = \frac{-1 \pm 9}{4}.$$

Thus $x = -\frac{5}{2}$ or $x = 2$.

(c) For $x^2 + 4x + 5 = 0$, take $a = 1$, $b = 4$, and $c = 5$. The quadratic formula gives

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{-4}}{2}.$$

The discriminant is $-4 < 0$, so the equation has **no real solutions**.

2. Factor the expressions

(a) Use the difference of squares:

$$x^2 - 4 = (x - 2)(x + 2).$$

(b) The numbers -2 and -3 multiply to 6 and add to -5 :

$$x^2 - 5x + 6 = (x - 2)(x - 3).$$

(c) Split the middle term and factor by grouping:

$$\begin{aligned} 2x^2 + x - 10 &= 2x^2 + 5x - 4x - 10 \\ &= x(2x + 5) - 2(2x + 5) \\ &= (2x + 5)(x - 2). \end{aligned}$$

The factors also confirm the two roots found in Question 1(b).

3. Rationalize the denominator

Multiply numerator and denominator by the conjugate $\sqrt{x+2}+3$:

$$\begin{aligned}\frac{x^2+4}{\sqrt{x+2}-3} &= \frac{(x^2+4)(\sqrt{x+2}+3)}{(\sqrt{x+2}-3)(\sqrt{x+2}+3)} \\ &= \frac{(x^2+4)(\sqrt{x+2}+3)}{(x+2)-9} \\ &= \boxed{\frac{(x^2+4)(\sqrt{x+2}+3)}{x-7}}.\end{aligned}$$

The denominator contains no square root. The numerator may remain factored.

Domain: The square root requires $x \geq -2$, and the original denominator is zero at $x = 7$. Both expressions therefore have domain

$$\boxed{[-2, \infty) \setminus \{7\}}.$$

On this domain, $\sqrt{x+2}+3 > 0$, so multiplying by the conjugate divided by itself is valid.

4. Simplification and domains

(a) Factor the numerator and denominator:

$$x^2+x-2 = (x+2)(x-1), \quad x^2-4x+3 = (x-1)(x-3).$$

Thus

$$f(x) = \frac{(x+2)(x-1)}{(x-1)(x-3)}, \quad x \neq 1, 3.$$

(b) Cancel the common factor $x-1$, which is nonzero on the domain of f :

$$f(x) = \frac{x+2}{x-3} \quad \text{for } x \neq 1, 3.$$

The simplified formula is $\boxed{g(x) = \frac{x+2}{x-3}}.$

(c) Taking each formula on its natural real domain, f and g are **not the same function**:

$$\text{dom}(f) = \mathbb{R} \setminus \{1, 3\}, \quad \text{dom}(g) = \mathbb{R} \setminus \{3\}.$$

They agree wherever f is defined, but $f(1)$ is undefined while $g(1) = 3/(-2) = -3/2$. Canceling a factor does not restore an excluded input to the original function.

Domain convention: If g is explicitly restricted to $x \neq 1, 3$, then it is the same function as f . A function includes its domain as well as its formula.