Quiz 04 – Version A Solutions

1. (5 points) For $x^2 + xy + y^2 = 3$, find dy/dx and evaluate it at $(1, 1)$.

Differentiate implicitly, using the product rule on xy:

$$2x + y + xy' + 2yy' = 0.$$

Collect the y' terms and solve:

$$(x + 2y)y' = -(2x + y), \qquad \boxed{y' = -\frac{2x + y}{x + 2y}}, \quad x + 2y \neq 0.$$

The point lies on the curve because $1^2 + 1 \cdot 1 + 1^2 = 3$. At $(1, 1)$,

$$\boxed{y' = -\frac{2(1) + 1}{1 + 2(1)} = -1}.$$

Rubric (5 points):

- **1 point:** Differentiate x^2 to obtain $2x$.
- **1 point:** Differentiate xy to obtain $y + xy'$.
- **1 point:** Differentiate y^2 to obtain $2yy'$.
- **1 point:** Collect terms and solve for y'.
- **1 point:** Substitute $(1, 1)$ and obtain slope -1.

2. (5 points) Find dy/dx for $y = x \cdot \arctan(2x)$.

Apply the product rule and the chain rule:

$$y' = 1 \cdot \arctan(2x) + x \cdot \frac{1}{1 + (2x)^2} \cdot 2$$

$$= \boxed{\arctan(2x) + \frac{2x}{1 + 4x^2}}.$$

This formula holds for every real x.

Rubric (5 points):

- **1 point:** Set up both terms of the product rule.
- **1 point:** Obtain the first term $\arctan(2x)$ from differentiating x.
- **1 point:** Use the arctangent derivative factor $1/(1 + (2x)^2)$.
- **1 point:** Include the inner derivative factor 2.
- **1 point:** Assemble the correct final derivative, retaining the factor x in the second term.

Grading notes: Accept equivalent correct methods and unsimplified expressions. Correct requested answers earn full credit even if intermediate work is omitted. For partial credit, award the steps supported by the work and carry arithmetic errors forward without deducting twice for the same error.