Quiz 02 – Version A Solutions

1. (5 points) Evaluate $\lim_{x\to 0} \dfrac{\sin 3x}{\tan 5x}$ using the important trigonometric limits.

Direct substitution gives the indeterminate form $0/0$. For nonzero x sufficiently close to 0, write tangent as sine divided by cosine:

$$\frac{\sin 3x}{\tan 5x} = \frac{\sin 3x \cos 5x}{\sin 5x} = \frac{3}{5}\left(\frac{\sin 3x}{3x}\right)\left(\frac{5x}{\sin 5x}\right)\cos 5x.$$

Since $3x \to 0$ and $5x \to 0$, the important sine limit gives

$$\frac{\sin 3x}{3x} \to 1, \qquad \frac{\sin 5x}{5x} \to 1, \qquad \frac{5x}{\sin 5x} \to 1.$$

Also $\cos 5x \to 1$ by continuity. Thus

$$\lim_{x\to 0} \frac{\sin 3x}{\tan 5x} = \frac{3}{5} \cdot 1 \cdot 1 \cdot 1 = \boxed{\frac{3}{5}}.$$

Rubric (5 points):

- **1 point:** A valid trigonometric rewrite of the quotient.
- **2 points:** Normalize the sine ratios (1 point) and obtain the correct scale factor $3/5$ (1 point).
- **1 point:** Apply the important trigonometric limits and continuity to evaluate the remaining factors.
- **1 point:** Complete the evaluation to obtain $3/5$.

2. (5 points) Use the IVT to show that $x^3 + 2x = 5$ has a solution in $[1, 2]$.

Let $f(x) = x^3 + 2x$. Since f is a polynomial, it is continuous on $[1, 2]$. Its endpoint values are

$$f(1) = 1 + 2 = 3, \qquad f(2) = 8 + 4 = 12.$$

The target value lies strictly between them: $3 < 5 < 12$. By the Intermediate Value Theorem, there exists $c \in [1, 2]$ such that

$$\boxed{c^3 + 2c = 5}.$$

In fact $c \in (1, 2)$ because neither endpoint gives the value 5. The IVT establishes existence; no exact solution is required.

Rubric (5 points):

- **1 point:** State continuity on $[1, 2]$, with a valid reason.
- **2 points:** Compute the endpoint values correctly (1 point each).
- **1 point:** Check that the target lies between the endpoint values, or that the corresponding zero-finding function changes sign.
- **1 point:** Invoke the IVT and conclude that a solution exists in $[1, 2]$.

Grading notes: Accept equivalent correct methods. Award partial credit for demonstrated steps. Carry an arithmetic error forward when later reasoning is correct; deduct for the same error only once. For Problem 1, a correct answer without work earns 1 point. For Problem 2, numerical approximation alone does not earn the IVT-conclusion point.