

Quiz 01 – Version A Solutions

1. (5 points) Find a so that f is continuous at 2, where

$$f(x) = \begin{cases} 3x - 1, & x < 2, \\ ax^2 + 1, & x \geq 2. \end{cases}$$

The one-sided limits and the function value are

$$\lim_{x \rightarrow 2^-} f(x) = 3(2) - 1 = 5, \quad \lim_{x \rightarrow 2^+} f(x) = 4a + 1 = f(2).$$

Continuity requires $5 = 4a + 1$, so $\boxed{a = 1}$. For this value, both one-sided limits and $f(2)$ equal 5; therefore $\lim_{x \rightarrow 2} f(x) = f(2)$.

2. (5 points) Evaluate $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4}$.

Direct substitution gives the indeterminate form $0/0$. For $x \neq 4$ near 4, multiply by the conjugate:

$$\begin{aligned} \frac{\sqrt{x+5} - 3}{x - 4} &= \frac{(\sqrt{x+5} - 3)(\sqrt{x+5} + 3)}{(x - 4)(\sqrt{x+5} + 3)} \\ &= \frac{x + 5 - 9}{(x - 4)(\sqrt{x+5} + 3)} = \frac{1}{\sqrt{x+5} + 3}. \end{aligned}$$

The simplified expression is continuous at 4, with nonzero denominator. Thus

$$\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4} = \frac{1}{\sqrt{9} + 3} = \boxed{\frac{1}{6}}.$$