MATH 31A: DIFFERENTIAL AND INTEGRAL CALCULUS
2026 FALL - Section 4
Hong Chen

MIDTERM EXAM 1
Time allowed: 60 minutes **Total: 100 points**

Your Name _____

The Last **Six** Digits of Your Student ID number | | | | | | |

By signing below, you confirm that you did not cheat on this exam.

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INSTRUCTIONS

- Please do **not** open this booklet until you are told to do so.
- Your cell phone must be turned off completely and stowed away where you cannot access it.
- No calculators, books, or notes are allowed.
- You are not allowed to access the Internet during the exam.
- If you have a question at any time during the exam, please raise your hand.
- Make sure to show all your work and justify your answers fully.
- Make sure to write legibly. **Illegible work will not be graded.**
- If you are using a pencil, make sure your writing is dark and easily visible.
- Any work you cross out will not be graded.
- Please write your answers in the space provided. You may use the back for scratch work only.

FRIENDS, GOOD LUCK! YOU'VE GOT THIS!

1. (30 points) Evaluate each limit, giving its finite value, $+\infty$, or $-\infty$ as appropriate. If none of these applies, write DNE. Justify your answers.

(a) (6 points)

$$\lim_{x \to -3} \frac{x^2 + 4x + 3}{2x + 6}$$

(b) (6 points)

$$\lim_{x \to \infty} \frac{3x + x^2 + 2}{x + 1}$$

(c) (6 points)

$$\lim_{t \to 2} \left(\frac{1}{t - 2} - \frac{4}{t^2 - 4} \right)$$

1. (continued)

(d) (6 points)

$$\lim_{\theta \to 0} \frac{2\sin(5\theta)}{\theta \cos(\theta)}$$

(e) (6 points) In this part, consider the function $f(x)$ defined below:

$$f(x) = \begin{cases} |x-5| & x > 2 \\ 3 & x = 2 \\ x^3 - 11 & x < 2. \end{cases}$$

Compute $\lim_{x \to 2^+} f(x)$ and $\lim_{x \to 2^-} f(x)$. Then compute $\lim_{x \to 2} f(x)$, or explain why it does not exist.

2. (20 points)

(a) (10 points) Complete the following three definitions:

- A function f has a *removable discontinuity* at $x = a$ if $\lim_{x \to a} f(x)$ exists and is finite, but

$$\lim_{x \to a} f(x) ____ f(a) \qquad \text{or} \qquad f(a) _____.$$

- A function f has a *jump discontinuity* at $x = a$ if $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exist and are finite, but

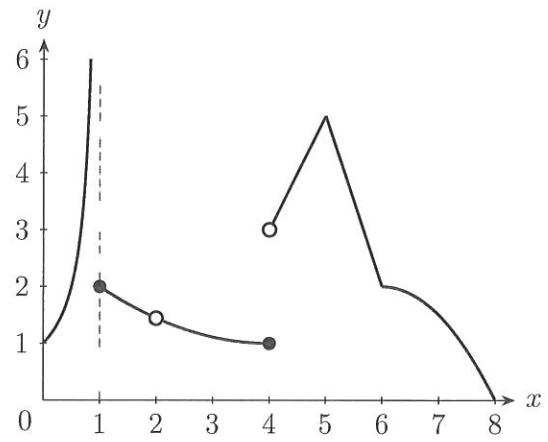
$$_____.$$

- A function f has an *infinite discontinuity* at $x = a$ if

$$\lim_{x \to a^-} f(x) = _____ \qquad \text{and/or} \qquad \lim_{x \to a^+} f(x) =$$

2. (continued)

(b) (10 points) Shown below is the graph of function f:



(In particular, $\lim_{x\to 1^-} f(x) = \infty$.)

Based on the graph shown, circle **TRUE** or **FALSE** for each statement. You do **not** need to justify your answers. Each statement is worth 1 point.

(1) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ **TRUE** **FALSE**

(2) f is both left- and right-continuous at $x = 2$. **TRUE** **FALSE**

(3) f has a removable discontinuity at $x = 4$. **TRUE** **FALSE**

(4) f has a vertical asymptote at $x = 1$. **TRUE** **FALSE**

(5) f is left-continuous at $x = 1$. **TRUE** **FALSE**

(6) $\lim_{x\to 6^+} f(x) = 2$ **TRUE** **FALSE**

(7) f is differentiable at $x = 5$. **TRUE** **FALSE**

(8) $\lim_{x\to 7} f(x) = f(7)$ **TRUE** **FALSE**

(9) $\lim_{x\to 6} \dfrac{f(x) - f(6)}{x - 6}$ does not exist. **TRUE** **FALSE**

(10) $f'(x) < 0$ for $0 < x < 1$. **TRUE** **FALSE**

3. (25 points) Let $f(x) = \dfrac{1}{3x+1}$.

(a) (5 points) Use the definition of the derivative to write a limit expression for $f'(1)$. Do not evaluate the limit.

$$f'(1) =$$

(b) (12 points) Evaluate the limit in part (a) to find $f'(1)$.

(c) (8 points) Find an equation of the tangent line to the graph of f at $x = 1$.

4. (25 points) Find the derivative of the following functions.

(a) (10 points) $f(x) = 2x^3e^x + 3x^2e^x + xe^2$.

(b) (15 points) $g(x) = \dfrac{xe^x}{x+1}$.