

MATH 170E Worksheet 1

Counting

Review.

- If every outcome is equally likely then, $\mathbb{P}(A) = \frac{|A|}{|\Omega|}$
- The number of permutations of a set of n elements is $n! := n(n-1) \cdots 2 \cdot 1$.
- The number of sequences of length k with elements from set of size n is n^k . The number of sequences with no repeated elements is $n(n-1) \cdots (n-k+1) = \frac{n!}{(n-k)!}$. With, it is n^k .
- The number of k -element subsets of an n -element set is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Problem 1. Let $\Omega = \{1, 2, \dots, 20\}$ and $\mathbb{P}$ be the uniform distribution on Ω . Find the probabilities that the outcome is (a) divisible by 3, (b) congruent to 1 modulo 4, and (c) divisible by 3 or congruent to 1 modulo 4.

Problem 2. A license plate has three letters followed by three digits. Letters and digits may repeat. Assume every plate is equally likely.

- (a) How many plates are possible?
- (b) What is the probability that its letters are all different and its digits are all different?

Problem 3. Let $n \geq 4$. Consider sequences (a_1, a_2, a_3, a_4) with entries in $\{1, \dots, n\}$.

- (a) Count all sequences, allowing repeated entries.
- (b) Count sequences with four distinct entries.
- (c) Count sequences with $a_1 < a_2 < a_3 < a_4$. Explain why the answer to (b) is $4!$ time the answer here.

Problem 4. From 10 students choose a committee of 4, then designate one member as chair and a different member as secretary. Count the outcomes in two ways: choose the committee first, or choose the officers first. Show that the two expressions agree.

Problem 5. Four digits are drawn independently and uniformly from $\{0, 1, \dots, 9\}$. You win if the sequence drawn consists of your four chosen digits, in some order. Find your probability of winning if you choose (a) 6, 7, 8, 9, (b) 6, 7, 8, 8, or (c) 7, 7, 8, 8. Explain how repeated digits change the number of winning sequences.