Math 170E - Fall 2026

Midterm Exam 1

Time: 60 minutes **Total: 100 points**

Fall 2026 course weights: Midterm 1 20%, Midterm 2 30%, Final 50%.

Name:

1. Write your name clearly on the cover. Exams without a name will not receive a grade.
2. Do not open this test booklet until you are directed to do so.
3. Write your answers on the **front** of the pages provided for each question. You may use the **back** for scratch work.
4. Give exact answers. Answers may be left as expressions involving binomial coefficients (e.g., $\binom{8}{3}$), fractions (e.g., $\frac{2}{7}$), and powers (e.g., $\left(\frac{2}{3}\right)^4$). Decimal approximations are not required.
5. If you cannot solve a problem completely, show as much relevant reasoning and work as you can. Partial credit will be awarded for correct intermediate steps, except in Question 1, which is graded on final answers only.

1. (25 pts) Suppose that for events C and D, we know that $\mathbb{P}(C) = 0.4$ and $\mathbb{P}(D) = 0.3$.

   Treat parts (a)–(e) separately: the additional assumptions in one part do not apply to the other parts.

   Each part will be graded solely on the accuracy of its final answer.

   (a) If C and D are independent, what is $\mathbb{P}(C \cup D)$?

   (b) If C and D are disjoint, what is $\mathbb{P}(C|D)$?

   (c) If $D \subset C$, what is $\mathbb{P}(C|D)$?

   (d) If $\mathbb{P}(C \cup D) = 0.5$, what is $P\left(C^c \mid D\right)$?

   (e) If $\mathbb{P}(C|D) = 0.7$, what is $P\left(C^c \mid D\right)$?

2. (10 pts) Let X be a random variable with the following probability mass function:

$$p_X(k) = \begin{cases} C|k|\,, & k \in [-3,3], \quad k \in \mathbb{Z} \\ 0\,, & \text{otherwise} \end{cases}$$

Find the constant C.

3. (30 pts) A man has five coins: two have heads on both sides, one has tails on both sides, and two are ordinary fair coins, each with one head and one tail. He selects one coin uniformly at random without inspecting it and uses that same coin throughout. Given the selected coin, successive tosses are independent.

   (a) He tosses the selected coin without looking at either face. What is the probability that the face pointing down is a head?

   (b) He then observes that the face pointing up on this first toss is a head, without looking at the face pointing down. Given this observation, what is the probability that the face pointing down is a head?

Part (c) continues on the next page.

Question 3 (continued)

(c) He tosses the same coin a second time without looking at the result. Given only the observation in part (b), what is the probability that the face pointing down on the second toss is a head?

4. (10 pts) Five distinct numbers are selected uniformly at random from $1, 2, \ldots, 40$, without regard to order. What is the probability that exactly three of the selected numbers are even?

5. (15 pts) Ten balls, labeled $1, \ldots, 10$, are placed into five boxes, labeled $1, \ldots, 5$. Each ball independently chooses one of the five boxes with equal probability.

   (a) (5 pts) Describe a sample space in which all outcomes are equally likely. State the probability of each outcome.

   (b) (5 pts) What is the probability that both box 1 and box 2 are empty?

   (c) (5 pts) What is the probability that boxes 1 and 2 contain exactly one ball in total?

6. (10 pts) A fair coin is tossed seven times independently. Given that at least five of the tosses are heads, what is the probability that the number of heads is a prime number?