

Math 32A — Week 1 Worksheet

Vectors in the Plane

Review: Set notation and vector notation

A. Set-theory basics.

- **Membership:** $x \in A$ means “ x is an element of A .” $x \notin A$ means “ x is not an element of A .”
- **Subset:** $A \subseteq B$ means every element of A is in B . If $A \subseteq B$ and $A \neq B$, we sometimes write $A \subsetneq B$.
- **Common sets:**

$$\mathbb{N} = \{1, 2, 3, \dots\}, \quad \mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}, \quad \mathbb{R} = \text{real numbers}.$$

- **Set-builder notation:** $\{x \in \mathbb{R} : \text{property}(x)\}$ is the set of real numbers x satisfying the property.
Example: $\{x \in \mathbb{R} : x^2 < 4\} = (-2, 2)$.
- **Union / intersection:**

$$A \cup B = \{x : x \in A \text{ or } x \in B\}, \quad A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

- **Difference:** $A \setminus B = \{x \in A : x \notin B\}$.

B. Vector basics (in $\mathbb{R}^2$).

- A vector in the plane with X -coordinate a , and Y -coordinate b is denoted by

$$v = \begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{R}^2.$$

- If $P = (x_1, y_1)$ and $Q = (x_2, y_2)$, then the vector from P to Q is

$$\overrightarrow{PQ} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}.$$

- **Addition/subtraction:**

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a + c \\ b + d \end{pmatrix}, \quad \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a - c \\ b - d \end{pmatrix}.$$

- **Scalar multiplication:** for $\lambda \in \mathbb{R}$,

$$\lambda \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \lambda a \\ \lambda b \end{pmatrix}.$$

- **Euclidian norm:**

$$\left\| \begin{pmatrix} a \\ b \end{pmatrix} \right\| := \sqrt{a^2 + b^2}.$$

- **Parallel vectors:** u and v are parallel if $u = \lambda v$ for some nonzero λ .
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1. **Column form from points.** Let $P = (2, 4)$ and $Q = (7, 1)$.

(a) Compute $\overrightarrow{PQ}$ in column form.

(b) Compute $\|\overrightarrow{PQ}\|$.

2. **Equality (equivalence) of vectors.** Let $A = (-1, 3)$, $B = (1, 3)$, $C = (5, -2)$, $D = (7, -2)$.

(a) Compute $\overrightarrow{AB}$ and $\overrightarrow{CD}$ in column form.

(b) Are $\overrightarrow{AB}$ and $\overrightarrow{CD}$ equal?

3. **Basic vector operations.** Let

$$u = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, \quad v = \begin{pmatrix} 4 \\ -1 \end{pmatrix}.$$

Compute:

(a) $u + v$

(b) $u - v$

(c) $2u - 3v$

4. **Scalar multiplication and magnitude.** Let $v = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(a) Compute $\|v\|$.

(b) Compute $2v$ and $(-\frac{1}{2})v$.

(c) Compute $\|2v\|$ and $\|(-\frac{1}{2})v\|$.

5. **Parallel vectors.** Decide whether the following pairs of vectors are parallel. If they are, find a nonzero scalar λ such that $u = \lambda v$.

$$(i) \ u = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \ v = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \qquad (ii) \ u = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \ v = \begin{pmatrix} -2 \\ 4 \end{pmatrix}.$$