SUMMARY of a book

Mark Burgin

FUNDAMENTAL STRUCTURES OF KNOWLEDGE AND INFORMATION: REACHING AN ABSOLUTE

Ukrainian Academy of Information Sciences, Kiev, 1997

As the history of science demonstrates, cognition of nature has brought investigators to a deeper understanding of those aspects of matter that are different in many aspects from common human experience. For example, the prominent physicist Werner von Heisenberg stated (1959) that atoms and elementary particles do not completely belong to the world of things and facts. They are more structures than things that are the ordinary phenomena of the everyday life.

Investigation of the regularities of the physical world made possible at first to suggest and then to demonstrate that there exists the structural level that is more basic than the physical level of nature. That is why, mathematics, which is dealing with this structural level, is so powerful in cognition of the most fundamental physical objects. It gives an exact answer to the question of E. Wigner (1960) why mathematics being so abstract and remote from nature is so efficient in science explicating regularities of nature. Moreover, the structural level provides our understanding of the phenomenon of knowledge and its most essential aspect, knowledge of structures.

Thus, existence of a new level of nature was discovered and its existence was grounded by experimental methods of modern science. In addition, it was demonstrated that any aspect of reality has its structural level. This level ties together nature, a human being, and society as their common existential and cognitive basis.

Finding of the world of structures made possible another essential discovery: the most fundamental entity called a fundamental triad or a named set. It has been introduced and studied in a sequence of publications of the author.

A fundamental triad has the following structure:

$$\textbf{Entity 1} \xrightarrow{\textbf{connection}} \textbf{Entity 2} \qquad (1)$$

or

$$\textbf{Essence 1} \xrightarrow{\textbf{correspondence}} \textbf{Essence 2} \qquad (2)$$

Each component of a fundamental triad plays its unique role and has a specific name: the Entity 1 (Essence 1) is called the *support*, the Entity 2 (Essence 2) is called the *reflector* (or the *set of names*) and the connection (correspondence) is called the *reflection* (or the *naming relation*) of the fundamental triad.

People meet triads constantly in their everyday life, as well as they can found them in very fundamental structures of the universe. As a consequence, the word "triad" has been used for a long time. For example, such prominent scientist as Levy Strauss found different important triads in ethnography and ethnology [10] while one of the best sociologists of the XX century Zimmel wrote about importance of special triads in sociology [15]. But the meaning of the word “triad” in all cases was different, and all triads (which were discussed or studied earlier) are special cases of the general structure of the fundamental triad (the futrad) which was introduced and considered in a sequence of publications (cf. [3-7]).

In mathematics theory of fundamental triads is developed as the theory of named sets. Many mathematical structures are particular cases of named sets. The most important of such structures are fuzzy sets (Zadeh, 1965; Zimmermann, 1991) and multisets (Aigner, 1979; Knuth, 1973) because they are both natural generalizations of the concept of a set and have many important applications. Moreover, any ordinary set is, as a matter of fact, some named set, and namely, a singlenamed set, i.e., such a named set in which all elements have the same name. Really, there are two familiar and natural ways of constructing sets: extensional and intentional. On the one hand, in the extensional approach, given a multiplicity of objects, some or all of these objects can be conceived together as forming a set. While doing so, we baptize this set by some name. The simplest name of a set has the form X or A. Then all elements from the set X have one common name "an element from X". This name discerns elements from X and all other entities. On the other hand, intentional method is a construction of elements that form the set. Than each element in this set is named, in some sense, and its name is the procedure (algorithm, program) by which the in question is constructed.

A visual representation of a named set is given in Figure 1.

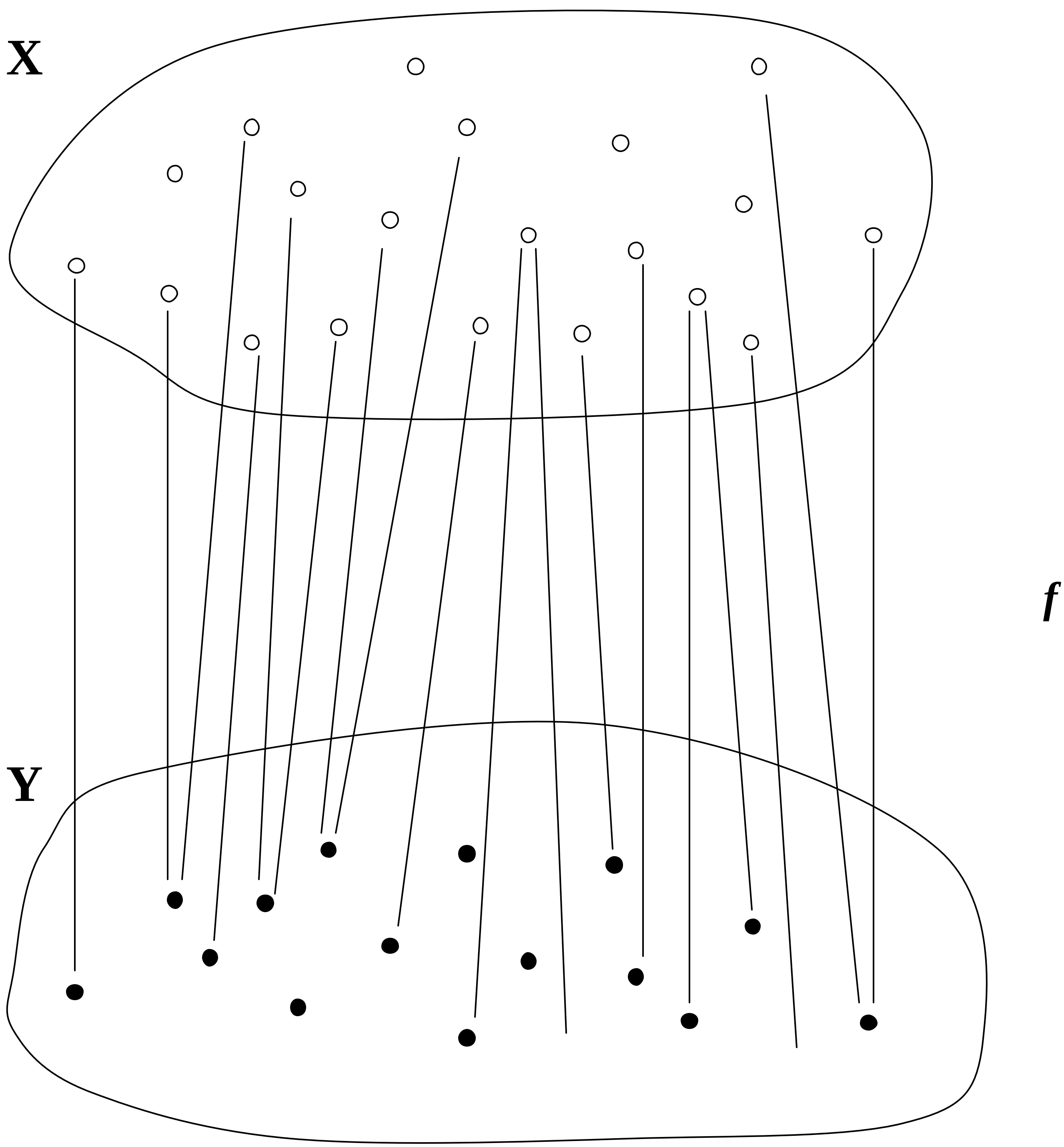

Fig. 1. A named set

Thus, we see that the fundamental triad **X** has the form (X, f, I) where X and I are some essences (entities, may be sets or classes) and f is a correspondence (connection) between X and I . Then X is the support, I is the reflector or the set of names and f is the reflection or the naming relation of the triad **X**.

It is necessary to remark that many kinds of triads are essentially indecomposable, indissoluble units. As a consequence, in such triads the objects X, f, I do not exist independently outside the triad which includes them.

Other kinds of triads are decomposable into three parts. But all these parts are either fundamental triads themselves or they consist of fundamental triads. It is an interesting problem whether such a decomposition may be infinite or it always ends after some number (may be very big but finite) of steps.

People meet fundamental triads constantly in their everyday life as well as they can found them in many fundamental structures of the universe. They can even see triads. For example, let us consider a rain. When people observe rain they see clouds (the support) from which streams of water (the naming correspondence or reflection) go to the earth (the reflector or set of names). Another example is given by a lightning. This named set has the same support and the reflector as the previous one. Its naming correspondence is the lightning which goes from clouds to the earth.

An interesting example of a named set which is visible (in some sense) to people is the following system: the two magnetic poles N and S (of a single magnet or of two separate ones) with the magnetic field f between these poles. This field f is a naming correspondence and can be represented as a series of lines linking the north (N) and south (S) poles of the magnet. As the direction of these lines is traditionally chosen from the north pole to the south pole, the lines had their source in the north pole N and their ends on the south pole S. Thus, N is the support, and S is the set of names of the considered named set (N, f, S).

If we consider nature as a whole, it is possible to see that it consists of a great variety of different systems: from systems of subatomic particles to the whole universe, which is also a system, from the blood system of a human being to an ecosystem. But what is a system? The following definition is given in the general system theory:

A system is a collection (a set) of elements with connections between them, or formally, a system **R** consists of a set *A* of elements and a set *C* of connections between them.

Binary set-theoretical relations usually represent these connections.

So, it is possible to see that the structure of **R** is a pair {*A*, *C*}. But taking into account that elements from *C* connect elements from *A*, we see that the structure of **R** is more adequately represented by the (fundamental triad) named set (*A*, *C*, *A*).

Another structural aspect of a system is explicated (Burgin and Karasik, 1975; Burgin, 1983) taking into consideration that elements from A are related to connections from *C*. It gives another named set representation of the system **R**: (*A*, ***r***, *C*). In it two elements *a* from *A* and *c* from *C* are corresponded to each other by the naming relation ***r*** (formally, it is designated by *a**r**c*) if and only if the connection *c* connects *a* with some (may be the same) element from *A*.

To have a more complete understanding of a system, it is necessary to consider the mereological approach (Meyen, 1977) on par with the set theoretical approach in the system theory. But, as it is demonstrated above, structures of systems (objects) in mereology are also triads. Mereology is a field in cognitive sciences, and the aim of mereology is investigation of different systems. Contrasting to the system theory, the mereological analysis is based not on elements and systems but on objects and their constituents that have another nature than elements of sets. Like a leg is not an element but a constituent of a human body. Although, constituents of an object are not separate elements in a general case, the structure of an object in mereology is also a named set (or a triad) because the constituents are connected with each other. These connections form the naming relation of the corresponding named set while all constituents constitute the support and the reflector of the same named set explicating the structure of an object in mereology. A major difference between the set theoretical and mereological approach is that connections as well as elements of supports and reflectors have different nature and properties.

Let us consider specific peculiarities of physical systems. One surprising feature of the physical world is that almost all of its great diversity can be described in terms of subatomic particles and four fundamental forces - the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force. Modern theories of these four forces are all based on the mathematical framework known as "quantum field theories". There are different kinds of such theories. Now the gauge field theory is the most popular in physical inquiries. The term "gauge" is related to the concept that measurements at different locations in space and time should not always give the same result in contrast to classical quantum field theories.

Mathematically, a field is characterized by assigning to each point of space a quantity that is intrinsically associated with it: a temperature, for instance, or a velocity, or a tensor, or a spinor of arbitrary rank. Thus, mathematically a field is a fundamental triad (M, f, C) consisting of a manifold M, a set of quantities C (numbers, vectors, spinors), and an assigning relation f. Thus, a field is a fundamental triad of special kind.

It is essential that, together with the specification of the field quantities, for the physical fields their transformation laws also have to be assigned under changes of the reference frame. Transformation laws are represented by such construction from the theory of named sets (fundamental triads) as a morphism of named sets. It is one of the main constructions of the theory of named sets (Burgin, 1984, 1987, 1990).

Moreover, quantums of fields are associated with subatomic particles and their constituents (quarks, gluons, preons etc.). So, subatomic particles are also represented by named sets.

Besides, in any system of such particles different types of interaction always exist between them. Modern physics distinguishes four types of interaction caused by the corresponding forces: weak, strong, electro-magnetic, and gravitational. Even an isolated subatomic particle acts on itself, and this action plays an important role in theories of elementary particles. Thus, any system of particles (may be consisting of a single element) has the structure of the fundamental triad (*P*, *I*, *P*) . In it *P* is the collection of the particles in question, and the relation I represents interaction between these particles.

According to modern physical theories, everything in nature is a system of physical fields and subatomic particles. As it is demonstrated, both these entities have the structure of a fundamental triad. In addition to this any transformation of fields and/or subatomic particles has the structure of a fundamental triad. Consequently, basing modern physical theories, we come to the conclusion that:

ANY NATURAL PHENOMENON HAS THE STRUCTURE OF SOME FUNDAMENTAL TRIAD (FUTRAD) OR OF SOME SYSTEM CONSISTING OF FUNDAMENTAL TRIADS (FUTRADS)

As a consequence, fundamental triads and their systems appear to be the basic objects of cognition, and the theory of fundamental triads helps to attain a new and profound understanding of nature's structure and behavior - with a refreshing and more simple (than before) way to describe it.

An interesting example of triads we can find even at the first moments of our universe existence. Shortly (~ 10^{-30} seconds) after the beginning of the process of the universe's expansion, the universe entered into a specific vacuum state called the "false vacuum". Calculations show extraordinary properties of the false vacuum. Among other things, it exerts an enormous effect on space itself. This effect is usually called "repulsion". But this is not repulsion in the ordinary sense of this word, it is in the sense of two entities like electric charges repelling each other in space. The repulsion in the false vacuum acts on space itself and forces it to expand at an enormous rate.

Consequently, we obtain the fundamental triad (*FV*, *Ac*, *SP*) where *FV* is the false vacuum, *SP* is the physical space and *Ac* is the action of *FV* on *SP*.

Another indication of the importance of fundamental triads for physics is the following fact. The two most fundamental areas of contemporary physics - quantum field theory and astrophysics - are reconstructed now basing on the superstring theory. This theory has five different versions that are united by the so called M-theory that eventually may include theory of gravitation. However, the main object in all cases is a mathematical model that is called a string. These strings are closed in additional implicit spatial dimensions and their structure is a fundamental triad for which its support is equal to its reflector.

The meaning and role of the fundamental triad evolved in a manner similar to those of a physical field. Although the field concept was originally thought to be a mere convenience, a way to visualize how forces can act between distant objects, the modern view of forces gives such fields a much more substantial role.

It is necessary to remark that the word "triad" has been used for a long time. But different authors corresponded different meanings to this word. Besides, all triads (which have been discussed or studied earlier) are either special cases of the general structure of the fundamental triad (the futrad) or are systems (like triangles) of fundamental triads.

A fundamental triad is new as a general concept. Moreover, an exact mathematical theory of fundamental triads (named sets) is created (Burgin, 1984;1986;1987; 1990; 1991) while all those objects which have been called triads before, were treated only on a conceptual level.

Theory of named sets provides new possibilities for set theory as a mathematical field. In the framework of theory of fundamental triads we can have a new understanding of different trends and directions of the mathematical set theory. Moreover, the main distinction among various trends in modern mathematics (such as classical, intuitionistic and constructive) and in its subfields - set theory (such as naive set theory of Georg Cantor, ZF-axiomatics, BG-axiomatics or descriptive set theory, (Fraenkel and Bar-Hillel, 1958) lies in those tools that are used for naming sets that may be used. The names for sets that are used have the form:

a) some letter (like *A* or *B*) or a linguistic expression *P(x)* in naive set theory;
b) a formal description in a set theoretical language using set variables, logical signs and other mathematical symbols (as in axiomatic theories like Zermelo-Fraenkel's (ZF) or Bernays-Godel (BG);
c) a descriptive representation using constructive (in some sense) operations;

d) an algorithm generating (computing) the elements from a constructive set or an identification algorithm of the elements from a recursive (decidable) set as it is in constructive or recursive mathematics.

Another example of named sets is provided by modern topology. Ordinary sets (or, as they are called in the theory of fundamental triads, singlenamed sets) constitute a basis for topology because a topological space is a set with a special structure that is called a topology. If we take instead of an ordinary set a general functional named set and induce the same structure on all three components of it, then we obtain such mathematical construction as a fiber that is very essential and efficient not only in mathematics but also in theoretical physics. Fibers are studied by a new and very important mathematical discipline - the theory of fibers.

Nevertheless, it is necessary to understand that if we consider named sets only as some constructions consisting of ordinary sets, we restrict ourselves to a great extent. As a matter of fact, such important class of mathematical objects as enumerations is a special case of named sets. Enumerations play an important role not only in mathematics but in the whole human culture, because any counting gives birth to some enumeration or its part.

The belief that any mapping or correspondence has to be a set theoretical construction is based on assumptions about universality of the theory of sets. It is not so. Set theory really provides foundations of mathematics but there are other approaches. One is an algorithmic or constructive approach. Another is an intuitionistic one. The notions of an algorithm or of a constructive process are central for them. In other words, a mapping *f* has to be an algorithm (according to the constructivism) or a constructive process (according to the intuitionism). Algorithms or processes are not sets. So, in this case the enumeration a is such a fundamental triad that does not consists entirely of sets. Namely, its reflection (naming relation) *f* is not a set theoretical entity.

Moreover, even in mathematics the supports and the reflectors of some named sets may be not sets but entities having other nature. Examples are given by such fields as the topology without points (Menger,1940) or mereology (Meyen, 1977).

Topology is a mathematical field in which objects are considered in a frame of continuous transformations. Thus, topology develops in mathematics the geometrical approach of the "Geschtalt" style of thinking which is related to the left hemisphere of human brains by modern neurophysiology.

The apparatus and methods of the mathematical part of the theory of named sets (fundamental triads) may be utilized rather effectively. This is demonstrated by the efficient applications of the theory of named sets (or triads) to the studies of knowledge systems (Burgin, 1991b; Burgin and Gladun, 1989; Burgin and Kuznetsov, 1992) , and of human cognition (Burgin and Gorsky, 1991; Burgin and Markov, 1991).

Speaking about ontological aspects of named sets (fundamental triads), it is interesting to compare the situation with the one which is connected with the concept of a physical field. Although the field concept was originally thought to be a mere convenience, a way to visualize how forces can act between distant objects, the modern view of forces gives the field a much more substantial role.

Moreover, it is demonstrated that fundamental triads are the most fundamental not only in nature but everywhere. Namely, the following law is valid.

ANY PHENOMENON HAS THE STRUCTURE OF SOME FUNDAMENTAL TRIAD (FUTRAD) OR OF SOME SYSTEM CONSISTING OF FUNDAMENTAL TRIADS (FUTRADS)

But what can be said about dyads? They also play an important role in different fields of human culture and human activity, for example, in cognition. At the same time, a person may have an impression that a dyad is simpler than a triad and, may be, even more elementary. Nevertheless, it is not the case.

Any dyad is a latent fundamental triad. Really, when two entities form a dyad ,then the necessary condition for existence of this dyad is existence of a tie between these entities. In such a way, we see that virtually the dyad constitutes a fundamental triad.

Sometimes Leibnitz proposed an idea of a monad as the most fundamental entity. Here, without opposing this idea, the conception of a fundamental triad having the precise structure is discussed, and it is argued that just such triads are the most elementary and the most basic entities. Thus, they are also fundamental objects of cognition providing as such very powerful means for cognitive processes.

It is necessary to remark that although fundamental triad is the utmost fundamental structure on the deepest level of the world including nature, there are other fundamental structures. On other levels and in different spheres of reality, such other structures may play even more important role than the fundamental triad. For example, in mathematics, when we base it on set theory, a special case of the fundamental triad - the so called sinlenamed set - becomes the most fundamental. When we take theory of categories as a foundation for mathematics, we encounter a situation in which the most fundamental is a complex system of fundamental triads. Mathematicians call this system a category.

A similar situation, we can find in science. Subatomic or elementary particles are the most fundamental for physics. At the same time, molecules are the most fundamental for chemistry. Biology begins with cells, and so on.

Theory of fundamental triads has three components: *mathematical* component (theory of named sets), *science* (including social sciences) component, and *philosophical* component.

In the book some elements of the theory of fundamental triads (named sets) as well as of the theory of chains of named sets are exposed and their applications to the knowsphere - the sphere of knowledge, intellect, and information are considered. The results are applied to various problems of computer science. For example, an exact distinction is obtained between data and knowledge making possible to extract more developed systems of data that are situated between data and knowledge.

Mathematical and science components provide grounds for the development of a new direction in philosophy - *neo-structuralism*, which is based on the philosophical component of the theory of fundamental triads. It is a synthesis of such well known philosophical fields as *structuralism* and *post-structuralism*. In its turn, *neostructuralism* is a part of such a field as the *structurology*, which includes also mathematics, which studies formalized structures.

This is a SUMMARY of the book

M. Burgin, FUNDAMENTAL STRUCTURES OF KNOWLEDGE AND INFORMATION:
REACHING AN ABSOLUTE, Ukrainian Academy of Information Sciences, Kiev, 1997

Selected R E F E R E N C E S

1. Aho, A.V., and Ullman, J.D. (1994) *Foundations of Computer Science*, Computer Science Press, New York
2. Aigner, M. (1979) *Combinatorial Theory,* Springer Verlag, Berlin/Heidelberg/New York
3. Atanasov, K. (1999) *Intuitionistic Fuzzy Sets*: *Theory and Applications*, Physica-verlag, Heidelberg/New York
4. Barrow, J.D. *Pi in the Sky: Counting, Thinking, and Being*, 1994
5. Beeson, M. (1985) *Foundations of Constructive Mathematics,* Springer-Verlag, Heidelberg
6. Bell, E.T. *Men of Mathematics*, Simon and Shuster, New York/London/Toronto, 1965
7. Berge, C. (1973) *Graphs and Hypergraphs*, North Holland P.C., Amsterdam/New York
8. Birkhoff G. and Lipson J.D. Heterogeneous algebras, *J. Combinatorial Theory*, 1970, No. 8
9. Bishop, E. 1967, *Foundations of Constructive Analysis*, McGraw-Hill, New York
10. Blehman, I.I., Myshkis, A.D., and Panovko, Ya.G. *Mechanics and Applied Logic*, Nauka, Moscow, 1983 (in Russian)
11. Boas, R.P. (1981) Can we make Mathematics Intelligible? The American Mathematical Monthly, 88, pp. 727-731

12. Bohm D. Wholeness and the Implicate Order, Routledge and Kegan Paul, 1980
13. Bourbaki, N. *Theorie des Ensembles*, Paris: Hermann, 1960
14. Bourbaki, N. *Topologie Generale,* Paris: Hermann, 1960
15. Burgin, M. On Greg's paradox in taxonomy, *Abstracts presented to the American Mathematical Society*, 1983, v. 4, No.3
16. Burgin, M. *Multicardinals*, Abstracts presented to the American Mathematical Society, 1983, v. 4, No. 7
17. Burgin, M.S. (1984) *'Named Sets and Information Representation'*, VII All-Union Conference on Mathematical Logic, Novosibirsk, 25 (in Russian)
18. Burgin, M.S. Representation of Algebraic and Topological Constructions by Means of Generalized Connected Relations, in "*Properties of Semigroups*," Leningrad, 1984, pp. 15-25 (in Russian)
19. Burgin M. Abstract theory of properties, in "*Nonclassical Logics*", Moscow, 1985 (in Russian)
20. Burgin M.S. *Functor Semantics in Categories of Named Sets*, in "*Rationality, Reasoning, Communication*", Kiev, 1987, pp. 183-198 (in Russian)
21. Burgin M.S.: 1988, '*Categories as Systems of Named Sets*', Abstracts of the American Mathematical Society, Vol.9. No. 2, p. 217
22. Burgin M.S. *Named Sets, General Theory of Properties and Logic*, Kiev, 1989, 32p. (in Russian)
23. Burgin M.S. *Knowledge in intellectual systems,* Conference on Intelligent Management Systems, Varna, 1989, pp. 281-286
24. Burgin M. Theory of Named Sets as a Foundational Basis for Mathematics, in "*Structures in Mathematical Theories*", San Sebastian, 1990, pp. 417-420
25. Burgin M.S. (1991) Categories, Semigroups and Named Sets, *Theory of Semigroups and* Applications, Saratov, v. 10, pp.7-16 (in Russian)
26. Burgin M.S. (1991a) Named Set Compositions in Categories, in "*Problems of Group Theory and Homological Algebra*", Yaroslavl, pp.39-52 (in Russian)
27. Burgin M.S. Algebraic Sructures of Multicardinal Numbers, in "*Problems of group theory and homological algebra*", Yaroslavl, 1992, pp. 3-20 (in Russian)

28. Burgin M.S. *Named sets as a mathematical apparatus for a data structures representation,* in “Problem Solving on Mathematical Models of Object Domains”, Kiev, 1992, pp. 13-21 (in Russian)
29. Burgin M. System Approach to the Concept of Time, *Philosophical and Sociological Thought*, 1992, No. 8 (in Russian and Ukrainian)
30. Burgin M. *On the way to the "Absolute": Triad is the most fundamental structure in human society,* Visnik of the Academy of Science of Ukraine, 1993, No. 5, pp. 29-35 (in Ukrainian)
31. *Information triads*, Philosophical and Sociological Thought, 1993, No. 7-8 (in Russian and Ukrainian)
32. Burgin M.S. Axiomatics of the Theory of Named Sets, *Theory of Semigroups and its Applications*, No. 11, Saratov, 1993, pp. 8-17 (in Russian)
33. Burgin M. (1993a) Triad as a Fundamental Structure in Human Culture, *Studia Culturologia*, v.2, pp. 51-63
34. Burgin M. The Phenomenon of Knowledge, *Philosophical and sociological Thought*, 1995, No. 3-4 (in Russian and Ukrainian)
35. Burgin M. Dynamic criteria of human health, *International Congress "Foundations of ecology and spiritual health of a human being*", Crimea, 1995 (translated from Russian)
36. Burgin M. Triad as a Fundamental Psychological Structure, *Mathematics and Psychology in the Pedagogical System* "Technical University", Odessa, 1996, pp. 6-7 (in Russian)
37. Burgin M. *Triad as a Way to mutual understanding,* Philosophical and Sociological Thought, 1996, No. 7/8 (in Russian and Ukrainian)
38. Burgin M. The Triad of Power, *Politological Encyclopedic Dictionary,* Kyiv, 1997 (in Ukrainian)
39. Burgin M. *Fundamental Structures of Knowledge and Information*, Kiev, AIS, 1997 (in Russian)
40. Burgin M. *Finite and Infinite*, in “On the Nature and Essence of Mathematics”, Appendix, Kiev, 1998, pp.97-108 (in Russian)

41. Burgin M. *On the Nature and Essence of Mathematics,* Kiev, AIS, 1998 (in Russian)
42. Burgin M. *Mirrors, or a Reflection of Culture*, Movies-Theater, No. 2, 1998, p. 39 (in Ukrainian)
43. Burgin M. *Interaction of Cultures in the Named Set Context*, Language and Culture, v.1, Kiev, 1998, pp. 35-40 (in Russian)
44. Burgin M. *Knowledge and Data in Computer Systems*, in Proceedings of the ISCA 17th International Conference "Computers and their Applications", International Society for Computers and their Applications, San Francisco, California, 2002, pp. 307-310
45. Burgin M. and Chunihin, A. (1997) *Named Sets in the Analysis of Uncertainty*, in "Methodological and Theoretical Problems of Mathematics and Information Sciences (Computer Science)", Kiev, pp. 72-85 (in Russian)
46. Burgin M. and Gatenbein, R.*Knowledge Discovery, Information Retrieval, and Data Mining*, in Proceedings of the ISCA 17th International Conference "Computers and their Applications", International Society for Computers and their Applications, San Francisco, California, 2002, pp. 55-58
47. Burgin, M. and Gladun, V. *Mathematical Foundations of the Semantic Networks Theory*, Lecture Notes in Computer Science, 1989, 364, pp. 117-135
48. *Mathematical models of semantic networks based on the named sets,* in "Decision Support Systems", Budapest, 1990, pp.81-96 (in Russian, in collaboration)
49. Burgin M. and Gorski, D. (1991) *Towards the construction of general theory of concept,* in "The Opened Curtain", Oulder/San Francisco /Oxford, pp. 167-195
50. Burgin, M. and Kuznetsov, V. (1991) Burgin M.S. *System analysis of scientific theory based on the named set idea,* in "Systems research. 1985", Moscow, 1986, pp. 136-160 (in Russian)
51. Burgin, M. and Kuznetsov, V. (1992) Fuzzy sets as named sets, *Fuzzy sets and systems*, v. 46, No. 2, pp. 189-192
52. Burgin M. and Milov Yu. The Existential Triad of World, in *Qualitative Structures Method: Theory and Practice,* S.-Petersburg, 1996 (in Russian)

53. Burgin M. and Milov Yu. Three dimensions of geopolitical and geosocial systems: community, region, and the world, Konferencja "*Wspolnotowosc, Regionalism, Globalism*", Abstracty, Warszawa, 1996
54. Burgin M. and Milov Yu. *Grammatical Aspects of Language in the Context of the Existential Triad Concept*, in "On the Nature and Essence of Mathematics", Appendix, Kiev, 1998, pp. (in collaboration)
55. Cai Wen (1984) Introduction of Extension Set, *Buzefal*, No. 19, pp. 49-57
56. Cantor, G. (1895) Beiträge zur Begrundung der transfiniten Mengenlehre, *Math. Ann.*, bd. 46, s. 481-512
57. Chang, C. C. and Keisler, H. J. (1990) *Model theory*. Third edition. Studies in Logic and the Foundations of Mathematics, 73. North-Holland Publishing Co., Amsterdam
58. Chunihin, A. (1997) *Fuzzy Sets and Multisets*: *Unity in Diversity*, in "Methodological and Theoretical Problems of Mathematics and Information Sciences (Computer Science)", Kiev, pp. 9-17 (in Russian)
59. Courant, R. and Robbins, H. (1969) *What is Mathematics*? Oxford University Press, New York/London/Toronto
60. Einstein, A. Address to the Prussian Academy of Science in Berlin, Jan 27, 1921
61. Fraenkel, A.A. and Bar-Hillel, Y. (1958) *Foundations of Set Theory*, North Holland P.C., Amsterdam
62. Gödel (1940) The Consistency of the Axiom of Choice and of the Generalized Continuum Hypothesis with the Axioms of Set Theory, *Annals of Math. Studies*, No. 3
63. Gogen, J.A. (1967) L-fuzzy Sets, *Journ. Math. Analysis Appl*., v. 18, pp. 145-174
64. Goldblatt, R. (1979) *Topoi: The Categorical analysis of Logic*, North-Holland P.C., Amsterdam/ New York/London
65. Gurevich Y. (1995) Evolving Algebra, In: *Specification and Validation Methods* (E.Börger, Ed.), Oxford Univ. Press
66. Halmos, P.R. (1962) *Algebraic Logic*, New York
67. von Heisenberg, W. (1959) *Physik und Philosophie*, Frankfurt am Main
68. Herrlich, H., and Strecker, G.E. *Category Theory*, Allyn and Bacon Inc., Boston, 1973
69. Hickman, J.L. (1980) A Note on the Concept of Multiset, *Bull. of the Australian Mathematical Society*, No. 22, pp. 211-217

70. Higgins, P.J. (1963) Algebras with a scheme of operators, *Math. Nachrichten*, v. 27, No. 1-2

71. Higgins, P.J. (1973) *Gropoids and categories*, North-Holland

72. Knuth, D. *The Art of Computer Programming*, v.2: *Seminumerical Algorithms*, Third Edition, Reading, Massachusetts: Addison-Wesley, 1997

73. Knuth, D. *The Art of Computer Programming*, v.3: *Sorting and Searching*, Second Edition, Reading, Massachusetts: Addison-Wesley, 1998

74. Lawvere, F.W. (1969) Andjointness in Mathematics, Dialectica, v. 23, pp. 281-296

75. Leblanc, L. (1962) Nonhomogeniuos Polyadic Algebras, *Proceedings of the American Mathematical Society*, v. 13, No. 1, pp. 59-65

76. Markov, A.A. On Constructive Mathematics, Transactions of the Mathematical Institute of the Academy of Sciences of the USSR, v. 67, 1962 (in Russian)

77. Markov, A.A. On the Logic of Constructive Mathematics, Moscow, Znaniye, 1972 (in Russian)

78. Matthiessen, G. A heterogeneous algebraic approach to some problems in automata theory, many-valued logic and other topics. *Contributions to general algebra (Proc. Klagenfurt Conf., Klagenfurt, 1978)*, pp. 193--211, *Heyn, Klagenfurt*, 1979

79. McVicar-Whelan, P.J. (1977) Fuzzy and multivalued logic, *7th International Symposium on multivalued logic*, N.C., pp. 98-102

80. Mendelson, E. *Introduction to mathematical logic*. Fourth edition. Chapman & Hall, London, 1997

81. Von Neumann, J. (1925) Eine Axiomatisierung der Mengenlehre, *Journ. für Math.*, v. 154, pp. 219-240

82. Peterson, J.L. *Petri Net Theory and the Modeling of Systems*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1981

83. Plotkin B.I. *Groups of Automorphisms of Algebraic Systems*, Moscow, Nauka, 1972 (in Russian)

84. Plotkin B.I. *Universal Algebra, Algebraic Logic, and Databases*, Moscow, Nauka, 1991 (in Russian)

85. Salii, V.N. (1965) Binary L-relations, *Izv. Vysh. Uchebn. Zaved., Matematika*, v. 44, No. 1, pp. 133-145 (in Russian)

86. Shepherdson and Sturgis (1963) Computability of Recursive Functions, *Journal of the ACM*, v. 10, No. 2, pp. 217-255

87. Shoenfield, J.R. Mathematical logic. *Association for Symbolic Logic, Urbana, IL; A K Peters, Ltd., Natick, MA*, 2001

88. Shönhage, A. (1980) Storage Modification Machines, *SIAM Journal on Computing*, v. 9, pp. 490-508

89. Stanley, R.P. *Enumerative Combinatorics*, Wadworth&Brooks/Cole, Monterey, California, 1986

90. Wigner, E. "The Unreasonable Effectiveness of Mathematics", *Communications in Pure and Applied Mathematics*, v. 13, 1960

91. Zimmermann, K.-J. *Fuzzy set theory and its applications*, Boston/Dordrecht/London, 1991