Math 33AH, Linear Algebra and Applications

Winter 2013

Homework 9 and Practice Final

Name:

Due: Friday, 3/15, before class

Problem 1: Compute an eigenbasis of $\mathbb{R}^3$ for the matrix

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 2 & 6 & -2 \\ 0 & 0 & 2 \end{pmatrix}.$$

(10 pts)

Problem 2: Use the method of least squares to find the optimal fit of the data points $(0, 2)$, $(1, 1)$, $(2, 4)$, $(3, 3)$ by a line in the x-y-plane. (10 pts)

Problem 3: Let V be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 0 \end{pmatrix}$$

Find an orthonormal basis of the orthogonal complement $V^\perp$ of V. (10 pts)

Problem 4: Recall that an $n \times n$-matrix A is called *nilpotent* if there exists a number $k \in \mathbb{N}$ such that $A^k = 0$.

a) What is the determinant of a nilpotent matrix A? Justify your answer! (3 pts)

b) Show that $\lambda = 0$ is an eigenvalue of every nilpotent matrix A. (2 pts)

c) Show that $\lambda = 0$ is the only eigenvalue of a nilpotent matrix A, i.e., if λ is an eigenvalue of A, then $\lambda = 0$. (5 pts)

Problem 5: Let A be an $n \times n$-matrix. Then A is called *orthogonal* if $A^T A = AA^T = I_n$; in other words, A is orthogonal if A is invertible and $A^{-1} = A^T$.

a) Show that the linear transformation associated with an orthogonal matrix preserves lengths of vectors; more precisely, show that if A is orthogonal, then $\|Ax\| = \|x\|$ for all $x \in \mathbb{R}^n$. (3 pts)

b) Show that if B is an $n\times n$-matrix satisfying $x^T By = x^T y$ for all vectors $x, y \in \mathbb{R}^n$ (here x and y are considered as column vectors), then $B = I_n$. (3 pts)

c) Prove the converse of (a): if $\|Ax\| = \|x\|$ for all $x \in \mathbb{R}^n$, then A is an orthogonal matrix. Hint: Consider $\|A(x + y)\|$ for $x, y \in \mathbb{R}^n$. (4 pts)