

Math 33AH, Linear Algebra and Applications  
Winter 2013  
Homework 3 and Practice Exam

Name:

Due: Friday, 2/1/13, before class



**Problem 1:** Find all solutions  $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$  of the system of equations

$$\begin{aligned}x_1 + 3x_2 - 4x_3 + 3x_4 &= 9, \\3x_1 + 9x_2 - 2x_3 - 11x_4 &= -3, \\4x_1 + 12x_2 - 6x_3 - 8x_4 &= 6, \\2x_1 + 6x_2 + 2x_3 - 14x_4 &= -12.\end{aligned}$$

(10 pts)



**Problem 2:** Show that the matrix

$$A = \begin{pmatrix} 1 & 2 & 2 \\ -1 & -1 & 1 \\ 3 & 5 & 4 \end{pmatrix},$$

is invertible, and compute its inverse  $A^{-1}$ .

(10 pts)



**Problem 3:**

a) Let  $V$  be a subset of  $\mathbb{R}^n$ . When is  $V$  called a *subspace* of  $\mathbb{R}^n$ ? Give a precise definition! (3 pts)

b) Consider the subset  $V$  of  $\mathbb{R}^4$  consisting of all vectors  $(x_1, x_2, x_3, x_4)$  in  $\mathbb{R}^4$  such that  $x_1 \geq -1$ , that is,

$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 \geq -1\}.$$

Is  $V$  a subspace of  $\mathbb{R}^4$ ? Justify your answer! (2 pts)

c) Suppose  $U$  and  $V$  are subsets of  $\mathbb{R}^n$ . Then the intersection  $U \cap V$  of  $U$  and  $V$  is defined to be the set of all vectors  $x$  in  $\mathbb{R}^n$  that belong both to  $U$  and  $V$ , that is,

$$U \cap V = \{x \in \mathbb{R}^n : x \in U \text{ and } x \in V\}.$$

Show that if  $U$  and  $V$  are subspaces of  $\mathbb{R}^n$ , then  $U \cap V$  is also a subspace of  $\mathbb{R}^n$ . (5 pts)



**Problem 4:**

a) Let  $A, B, C$  be  $(2 \times 2)$ -matrices. Suppose

$$A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix},$$

and

$$A^{-1}BA = C.$$

Find the matrix  $B$ . Hint: First express  $B$  in terms of  $A, A^{-1}, C$ . (5pts)

b) Let  $A$  be an  $(n \times n)$ -matrix. Show that if  $A^{10} = I_n$ , then the only solution  $x \in \mathbb{R}^n$  of the equation  $Ax = 0$  is given by  $x = 0$ . (5pts)



**Problem 5:** An  $(n \times n)$ -matrix  $A$  is called *nilpotent* if there exists  $k \in \mathbb{N}$  such that  $A^k = \mathbf{0}$ . Here  $\mathbf{0}$  denotes the  $(n \times n)$ -matrix whose all entries are equal to 0.

a) Give an example of a  $(3 \times 3)$ -matrix  $A \neq \mathbf{0}$  that is nilpotent. (5 pts)

b) Show that no nilpotent matrix  $A$  is invertible.

Hint: Argue by “contradiction”; that is, assume that there exists an invertible nilpotent matrix  $A$ . Derive a consequence that you know is false and conclude that your hypothesis (namely, that there exists an invertible nilpotent matrix) must also be false. (5 pts)