Advanced 2

Tiling with Integers

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Instructor's Handout

Problem 1:
Suppose that we have a rectangle R which can be tiled by smaller rectangles $r_1, \ldots, r_n$ such that each r_i has *at least one integer side length*.

Show that R has at least one integer side length.

Solution

Hint: try to find a function which integrates to 0 over a rectangle if any only if said rectangle has at least one integer side length.
$f(x, y) = e^{2\pi i(x+y)}$ works.
You can also make $f(x, y) = \sin(2\pi x)\sin(2\pi y)$ work if you shift R so that it is not centered on the origin.