**NEWTON’S METHOD**

**Review Problems**

**Exercise 1. (Mean Value Theorem):** Show that there exists some \( c \) in the given interval such that \( f'(c) \) satisfies the relationship described.

(i) \( f(x) = \sqrt{x} \) \[9, 25\] \( f'(c) = \frac{1}{6} \)

(ii) \( f(x) = x - \sin \pi x \) \([-1, 1]\) \( f'(c) = 1 \)

(iii) \( f(x) = (x - 1)(x - 3) \) \([1, 3]\) \( f'(c) = 0 \)

For more practice, you can find a specific \( c \) satisfying the relationship.

**Exercise 2. (Implicit Differentiation):** Give the equation for the tangent line of the given curve at the given point.

(i) \( xy + x^2y^2 = 6 \) at the point \((2, 1)\)

(ii) \( x^2 + \sin y = xy^2 + 1 \) at the point \((1, 0)\)

(iii) \( 2x^{1/2} + 4y^{-1/2} = xy \) at the point \((1, 4)\)

(iv) \( \sin(2x - y) = \frac{x^2}{y} \) at the point \((0, \pi)\)

**Exercise 3. (Extrema):** Find all local extrema of the following functions

(i) \( f(x) = \frac{1}{\sin x + 4} \)

(ii) \( f(x) = 9x^{7/3} - 21x^{1/2} \)

(iii) \( f(x) = 3x^4 - 6x^3 + 6x^2 \)

(iv) \( f(x) = \sin(x) \cos(x) \) (potential challenge)

**Exercise 4. (Concavity):** Determine the intervals on which the function is concave up or down and find the points of inflection.

(i) \( y = 10x^3 - x^5 \)

(ii) \( y = (x - 2)(1 - x^3) \)

(iii) \( y = x^{7/2} - 35x^2 \)

(iv) \( f(x) = \frac{x^3}{1+x} \)

(v) \( f(x) = \tan(x) \) (potential challenge)

**Newton’s Method**

Newton’s Method is a way of using the derivative of a function \( f(x) \) to numerically find (or approximate) the roots \( f(x) = 0 \). It does this by starting with a guess point \( x_0 \), assuming that \( f(x) \approx f(x_0) + f'(x_0)(x - x_0) \)
and then using this to determine a next best guess \( x_1 \). Iterating this process, we get better and better guesses \( x_0, x_1, x_2, \ldots \). More definitively, the process is defined as

\[
\begin{align*}
\textbf{Theorem. Newton’s Method:} & \quad \text{To approximate a root of } f(x) = 0, \\
\text{ (1) Choose an initial guess } x_0 \text{ (close to the desired root if possible)} \\
\text{ (2) Generate successive approximations } x_1, x_2, \ldots \text{ where} \\
& \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\end{align*}
\]

As a good rule of thumb, if \( x_n \) and \( x_{n+1} \) agree to \( m \) decimal places, then you can usually safely assume that \( x_n \) agrees with a root to \( m \) decimal places.