Course Announcement

Valued Fields: Algebra and Model Theory

Math 223M, Fall Quarter 2017

MWF 10 am–10:50 am, MS 5148

Instructor. Matthias Aschenbrenner (`matthias@math.ucla.edu`)

Office hours. by appointment; MS 5614.

Description. The concept of a valuation, which arose early in the 20th century in an attempt to better understand the then-newfangled concept of p-adic number, now plays an important role not only in number theory but also in commutative algebra and various flavors of geometry: algebraic, semialgebraic, rigid analytic, tropical, . . . , to name a few. The general theory of (Krull) valuations is not usually part of the graduate school curriculum in algebra, but is crucial for applications of model theory to these fields. The aim of this course is to provide a thorough introduction to valued fields, with a particular emphasis on their model theory, leading up to the classical results by Ax-Kochen-Ershov and Macintyre on the model theoretic properties of the p-adic numbers. This should prepare students to be able to study more recent developments, such as those of Cluckers, Denef, Hrushovski, Kazhdan, Loeser and others on motivic integration. I will try to make the course accessible for those with a basic knowledge of graduate algebra (groups, rings, fields) and some logic (on the level of 220A). If in doubt about your background, ask me.

References. My main references for this class will be:

M. Aschenbrenner, L. van den Dries, J. van der Hoeven, *Asymptotic Differential Algebra and Model Theory of Transseries,* Annals of Mathematics Studies, vol. 195, Princeton University Press, Princeton, NJ, 2017.

L. van den Dries, *Lectures on the model theory of valued fields,* in: D. Macpherson, C. Toffalori (eds.), *Model Theory in Algebra, Analysis and Arithmetic,* pp. 55–157, Lecture Notes in Mathematics, vol. 2111, Springer, Heidelberg, 2014.

A. Prestel, P. Roquette, *Formally p-adic Fields,* Lecture Notes in Mathematics, vol. 1050, Springer-Verlag, Berlin, 1984.

Here are some additional sources:

O. Endler, *Valuation Theory,* Springer-Verlag, 1972.

A. J. Engler, A. Prestel, *Valued Fields,* Springer Monographs in Mathematics, Springer-Verlag, Berlin, 2005.

P. Ribenboim, *Theorie des Valuations,* Les Presses de l'Université de Montréal, 1964.

P. Ribenboim, *The Theory of Classical Valuations,* Springer-Verlag, 1999.

O. Schilling, *The Theory of Valuations,* American Mathematical Society, 1950.