

Problem Set 7  
Due Friday, May 25.

*Real Analysis*

Math 131A, Spring Quarter 2018

1. Let  $S \subseteq \mathbb{R}$ . One says that an element  $x_0$  of  $S$  is *isolated* if there is an  $\varepsilon > 0$  such that  $(x_0 - \varepsilon, x_0 + \varepsilon) \cap S = \{x_0\}$ . Show that every function  $f: S \rightarrow \mathbb{R}$  is continuous at each isolated  $x_0 \in S$ .
2. Let  $g: S \rightarrow \mathbb{R}$  be continuous at  $x_0 \in S$ , and suppose  $g(x_0) \neq 0$ .
  - (a) Show that there is an open interval  $I$  such that  $x_0 \in I$  and  $g(x) \neq 0$  for each  $x \in I \cap S$ . (Distinguish between the case where  $x_0$  is isolated and non-isolated.)
  - (b) Show that  $1/g: I \cap S \rightarrow \mathbb{R}$  is continuous at  $x_0$ .
3. Do problems 17.5, 17.6, 17.10, 17.12, 17.13, 17.14 in the textbook.