Problem Set 2
Due Friday, April 20.

Real Analysis

Math 131A, Spring Quarter 2018

1. Let K be a field, i.e., a set equipped with two operations $+$, $\cdot$ on K satisfying the axioms (A1)–(A4), (M1)–(M4) and (DL) stated in class.
   (a) Show that there is only one element 0 of K satisfying the property in (A3).
   (b) Show that there is only one element 1 of K satisfying the property in (M3).
2. Consider the subset $K := \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ of $\mathbb{R}$.
   (a) Show that $0, 1 \in K$, and if $r, s \in K$, then $r + s$ and $r \cdot s$ also belong to K.
   (b) Verify that equipping K with the usual addition and multiplication of real numbers, restricted to K, turns K into a field.
   (c) Show that there exists a binary relation $\leq$ on K so that K becomes an ordered field. Extra credit: can you find two distinct such relations?
3. Do problem 3.5 in the textbook.
4. Do problems 4.1–4.4 for (a), (b), (k), (u), (v), as well as 4.14, 4.15, in the textbook.