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Nonlinearity management in optics

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We study nonlinearity management in optics by investigating the propagation of localized pulses and plane waves in a layered, cubically nonlinear (Kerr) medium that consists of alternating layers of glass and air. We show that such nonlinearity management delays the blow-up/collapse of pulses and leads to a band structure of modulationally unstable regions for plane waves. We find excellent agreement between experiments, numerical simulations, and theory.

1 Introduction

In optics, the nonlinear Schrödinger (NLS) equation models propagation of pulses in optical fibers or beams in optical media [1, 2]. It takes into account group-velocity dispersion (GVD) and diffraction, in addition to the intensity-induced change of the medium's refractive index. In the case of pulses in optical fibers, "dispersion management" refers to the implementation of an optical link in which GVD is periodic in the propagation variable. In this paper, we focus on the propagation of beams in layered nonlinear (Kerr) media, studying "nonlinearity management" – which entails the periodic variation of the nonlinearity with respect to the evolution variable [3]. In particular, we provided a brief overview of the first experimental implementation of nonlinearity management in an optical system composed of alternating layers of glass and air [4–6].

2 Results

We used an amplified femtosecond laser system in order to achieve the necessary intensity for optical nonlinearities in glass. The pertinent NLS equation, adapted to our experimental setting, takes the dimensionless form

$$iu_\zeta = -\frac{1}{2}\nabla_\perp^2 u - |u|^2 u\,, \quad 0 < \zeta < \tilde{l}\,(\text{glass})\,, \qquad iu_\zeta = -\frac{1}{2}\frac{\eta_0^{(1)}}{\eta_0^{(2)}}\nabla_\perp^2 u - \frac{\eta_2^{(2)}}{\eta_2^{(1)}}|u|^2 u\,, \quad \tilde{l} < \zeta < \tilde{L}\,(\text{air})\,, \tag{1}$$

where glass of scaled length $\tilde{l}$ alternates with air of scaled length $\tilde{L} - \tilde{l}$. In Eq. (1), space is rescaled by the wavenumber, $(\xi, \eta, \zeta) = k^{(1)} * (x, y, z)$, and the electric field envelope is scaled using $u = (n_2^{(1)}/n_0^{(1)})^{1/2}E$, where the superscript (j) denotes the medium ($j = 1$ for glass and $j = 2$ for air). Our numerical simulations consider radially symmetric solutions of Eq. (1), an approximation supported by the experimental data.

Figure 1 shows a comparison between the dynamics of the beam FWHM (full width at half-maximum) as a function of the propagation distance for our experiments and numerical simulations; the FWHM of the laser beam is measured as it propagates through alternating layers of glass and air [4]. The beam power is $P = 5.9P_c$, where P_c is the critical power for collapse in a self-focusing medium. It is readily observed that, without the help of any lossy mechanisms, the beam diameter is sustained for a much longer propagation in the layered medium than in either air (in which the beam width diverges) or glass (in which the beam undergoes collapse).

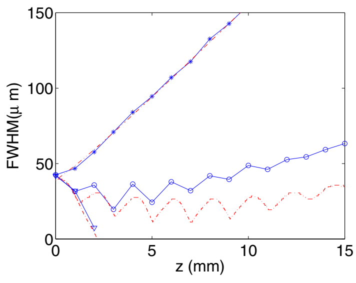

Fig. 1 [Color online] Beam FWHM (in μm) as a function of propagation distance (in mm) in the layered medium. The respective experimental results are denoted by triangles, circles, and stars (the solid curves are visual guides). The corresponding FWHMs computed from Eq. (1) are shown by the thin dash-dotted curves.

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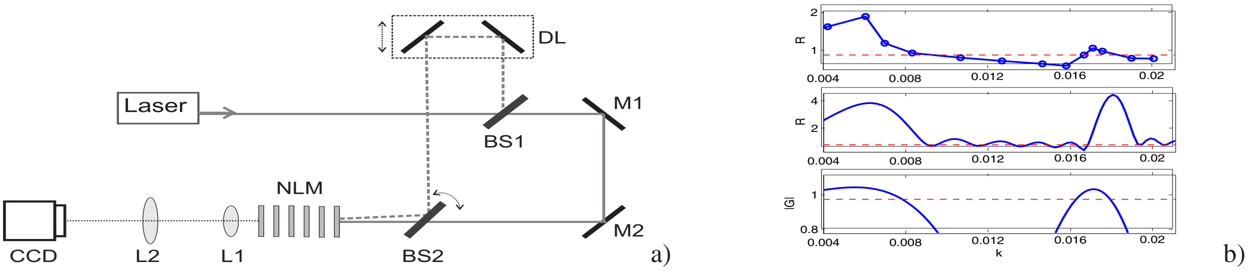

Fig. 2 [Color online] (**a**) Setup for MI experiments. BS1 and BS2 are beam splitters, DL is a variable delay line, M1 and M2 are mirrors, NLM is the layered nonlinear medium, and L1 and L2 are lenses. (**b**) Comparison of experimental (top), numerical (middle), and analytical (bottom) MI bands (for a layered medium consisting of 1 mm glass sandwiching 2.1 mm of air) as a function of the dimensionless wavenumber k. For the diagnostics R and $|G|$ (defined in the text), values larger than 1 correspond to MI.

We also studied modulational instability (MI) in the layered medium. MI is a destabilization mechanism for plane waves, leading to delocalization (localization) in momentum (position) space and the formation of localized (solitary-wave) structures [2, 7]. MI has been studied in many physical settings, including fluid dynamics (where it is also called the "Benjamin-Feir instability") [7], nonlinear optics [1], and atomic physics [8]. It is also arises in connection to the emergence of bright solitary waves (and trains thereof).

MI was originally analyzed in uniform media (predominantly in the NLS framework), where a focusing nonlinearity leads to MI for sufficiently large plane-wave amplitudes (for a given wavenumber) or sufficiently small wavenumbers (for a given amplitude) [1]. In other words, there is only a single band of modulationally unstable wavenumbers. However, periodicity in the propagation variable leads to multiple MI bands, as the perturbation of the plane wave solution to Eq. (1) satisfies a Hill equation [5, 6]. In the case of layered Kerr media, the nonlinearity is piecewise constant, and the Hill equation becomes the Kronig-Penney model known from solid-state physics [9]. The Floquet exponent ω satisfies

$$\cos(\omega\tilde{L}) = -\frac{s_1^2+s_2^2}{2s_1s_2}\sin(s_1\tilde{l})\sin[s_2(\tilde{L}-\tilde{l})] + \cos(s_1\tilde{l})\cos[s_2(\tilde{L}-\tilde{l})] \equiv G(\bar{k})\,, \tag{2}$$

where s_1^2 and s_2^2 are functions of the nonlinearity coefficients, the dispersion coefficients, and the amplitude of the plane wave. The condition $|G(\bar{k})| \leq 1$ implies stability and $|G(\bar{k})| > 1$ leads to MI. For the experiments and numerical simulations, we take the Fourier transforms of the initial plane wave and the wave after propagation and examine the relative sizes of the peaks at the perturbation wavenumber. The new peak is a factor R times the old peak, so MI occurs if $|R| > 1$. Figure 2**a** shows the setup for our experiments (see Refs. [5, 6] for details). Figure 2**b** shows the experimental, numerical, and theoretical results for the MI bands for a layered Kerr medium with 1 mm glass layers sandwiching 2.1 mm air gaps. The locations agree *quantitatively* even though there are no fitting parameters [5, 6].

3 Conclusions

We used layered optical media to provide the first experimental implementation of nonlinearity management in the NLS equation. Such management can be used to delay pulse collapse or dispersion by using two focusing media with a method that is lossless in principle. We obtain very good agreement with NLS simulations without any free parameters. We also studied modulational instability in the layered media, obtaining excellent quantitative agreement between experiments, numerical simulations, and theory for the locations of the instability bands (again, without free parameters). These results suggest a variety of extensions not only in the present setting of layered Kerr media but also in nonlinearity-managed Bose-Einstein condensates [3, 8], whose mean-field dynamics is also governed by NLS equations.

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