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ABSTRACT

We study the spreading dynamics of content on networks. To do this, we use a model in which content spreads through a bounded-confidence mechanism. In a bounded-confidence model (BCM) of opinion dynamics, the agents of a network have continuous-valued opinions, which they adjust when they interact with agents whose opinions are sufficiently close to theirs. Our content-spreading model, which one can also interpret as an independent-cascade model, introduces a twist into BCMs by using bounded confidence for the content spread itself. We define an analog of the basic reproduction number from disease dynamics that we call an *opinion reproduction number*. A critical value of the opinion reproduction number indicates whether or not there is an "infodemic" (i.e., a large content-spreading cascade) of content that reflects a particular opinion. By determining this critical value, one can determine whether or not an opinion dies off or propagates widely as a cascade in a population of agents. Using configuration-model networks, we quantify the size and shape of content dissemination by calculating a variety of summary statistics, and we illustrate how network structure and spreading-model parameters affect these statistics. We find that content spreads most widely when agents have a large expected mean degree or a large receptiveness to content. When the spreading process slightly exceeds the infodemic threshold, there can be longer dissemination trees than for larger expected mean degrees or receptiveness (which both promote content sharing and hence help push content spread past the infodemic threshold), even though the total number of content shares is smaller.

Although most content does not spread far on social media, occasionally some content "goes viral" and rapidly reaches many people.[1,2] When content with misinformation or disinformation spreads widely, it has become common to use analogies with disease spread and state that there is an *infodemic*,[3–5] which is a portmanteau of the words "information" and "epidemic." In the present paper, we examine infodemics in a model of content spread that uses a bounded-confidence mechanism. We extend the analogy between the spread of content and the spread of infectious diseases by defining an *opinion reproduction number*, which is inspired by the basic reproduction number of disease dynamics.[6,7] By examining whether or not the spreading process is below or above a critical opinion reproduction number (i.e., an *infodemic threshold*), one can examine whether content dies out or goes viral. Our investigation complements branching-process approaches that examine whether content spread tends to locally magnify or contract with time.[8] In our bounded-confidence model of content spread, we quantify the size and shape of content dissemination using a variety of summary statistics, and we illustrate how network structure and spreading-model parameters affect these statistics in configuration-model networks.

I. INTRODUCTION

Given the enormous scale and impact of human interactions on social media, it is critical to study the collective dynamics that arise in these systems.[9] Content on social media can take many forms, including scientific facts, other forms of information, hyperlinks to Web pages, pictures, memes, and misinformation and disinformation.[10–12] Once created, content can then be magnified by human users, bots, and other accounts, creating an online ecosystem that is filled with misinformation, disinformation, and "echo chambers,"[13–17] even with intentions to share only accurate information.[18] Content sharing by people is ubiquitous and important for social connection, and such sharing can help create and reinforce shared understanding.[19] Most content does not spread to many people, but some online content does spread very far (i.e., it "goes viral") in large content-spreading cascades.[1,2] A variety of other social phenomena, including emotions and behaviors, can also spread as "social contagions" on networks.[20–24]

When misinformation or disinformation spreads very widely, it has become reasonably common to state that there is an *infodemic*.[3–5] The World Health Organization (WHO) gives a much more specific definition of the term "infodemic" (which is a portmanteau of the words "information" and "epidemic") in the context of infectious diseases:[25,26] "An infodemic is too much information including false or misleading information in digital and physical environments during a disease outbreak." Using both the rigid WHO definition and the looser notion of particularly low-quality content going viral, it is recognized widely that the COVID-19 pandemic has had accompanying infodemics.[27–29] Analogously to the spread of infectious diseases, "superspreader" social-media accounts have played an important role in COVID-19 infodemics.[30]

Developing a thorough understanding of misinformation, disinformation, and their impact requires a broad view of the problem of "fake news" that also entails proper understanding of misinformation and its effects.[31,32] This view needs to encompass disinformation (which is intentionally incorrect), misinformation (which is incorrect, but perhaps unintentionally), biased and misleading information (which may not be factually incorrect), and the production and amplification of such content, including by mainstream news organizations.[16] There has been much empirical research on misinformation in areas such as computational social science and allied disciplines.[12] Importantly, modeling efforts can also play a significant role in mitigating the harmful effects of misinformation and disinformation.[33] In particular, Juul and Ugander[2] suggested that focusing on reducing the "infectiousness" of information and conducting theoretical analyses of spreading processes may be very helpful for limiting the spread of misinformation and disinformation.

One strategy to gain insight into the mechanisms that underlie observations in social-media systems is by studying mathematical models of opinion dynamics on networks.[34–36] Opinion models take a variety of forms. The nodes of a network represent agents, and the edges between agents indicate social and/or communication ties between agents. The opinions of the agents can take either discrete values (e.g., $+1$ or -1) or continuous values (e.g., in the interval $[-1, 1]$, with -1 representing the most liberal opinion and $+1$ representing the most conservative opinion). When two (or more) adjacent agents interact, one or more of them updates their opinion according to some rule. One popular type of opinion model is a *bounded-confidence model* (BCM),[37–39] in which agents have continuous-valued opinions and interacting agents compromise their opinions by some amount if and only if their opinions are sufficiently close to each other. There have been numerous studies of BCMs, which have been generalized in many ways. Recent studies have incorporated phenomena such as media outlets with fixed opinions,[40] polyadic interactions (i.e., interactions between three or more agents),[41,42] noise,[43] asymmetric confidence bounds,[44] costs of opinion changes,[45] agents with heterogeneous activity levels,[46] smooth interaction kernels (in the form of sigmoidal functions) to describe how agents influence each other,[47] opinion repulsion,[48] homophilic adaptivity of network structure,[49] dynamics with "no one left behind,"[50] and adaptive confidence bounds.[51]

Another line of modeling research is the study of spreading dynamics on networks.[22,52] Research on spreading dynamics often uses ideas from percolation theory.[53,54] The study of spreading processes on networks includes research on social dynamics,[55] disease dynamics,[56] and how they affect each other.[57] Much research on so-called "social contagions" has been inspired by the rich tradition of scholarship on biological contagions, although researchers argue about whether and how much social phenomena spread in a manner that resembles disease spread.[58–64] Nevertheless, despite the differences between disease spread and the spread of information and other content, it is worthwhile to explore parallels between them. Whether one is considering a disease or online content, some things spread very far before dissipating and others die out rapidly.[2] Indeed, people even say that online content that spreads very far has "gone viral."

In some models of social phenomena, such as simplistic cascade models and any other models in the form of a branching process,[65] one can study whether the spread of content tends to magnify or contract locally as a function of time by computing branching numbers,[8] which indicate the mean number of offspring (e.g., the mean number of reposts as a function of time) of a piece of content. This is a simplification of empirical content spread on social media, where it is common to trace the spreading paths of posts, post boosting, and post commenting and quoting using dissemination trees.[1,10,66,67] There are also other ways to calculate local magnification and contraction in spreading processes.[68] Analogously to studies of content spread in social systems, when researchers study compartmental models of disease spread, it is traditional to calculate reproduction numbers to examine whether or not a disease dies out.[7] The most standard type of reproduction number is the basic reproduction number, which measures how many infections occur, on average, from one infected node in a population in which all other nodes are susceptible to infection.[6,7] Compartmental models have also been employed in studies of opinion dynamics, and one can then calculate a basic reproduction number to examine whether or not content goes viral.[69] Additionally, in the study of disease spread, researchers have used branching-process theory in concert with maximum-likelihood estimation to estimate basic reproduction numbers from data.[70]

In the present paper, we combine ideas from opinion models and percolation-inspired spreading models to study the mean "infectiousness" of content in a model that uses a bounded-confidence mechanism for content spread.[71] In our content-spreading model, which one can also interpret as an independent-cascade model,[72] we use a bounded-confidence mechanism to determine whether or not an agent is receptive to content. If an agent is receptive, it spreads content without modifying it, rather than compromising its opinion as in a standard BCM. Our model takes the form of a threshold model[55] and has an opinion-update rule that is reminiscent of bond percolation.[53,73,74] In standard bond percolation, an edge of a network is preserved with probability p and is removed with probability $1 - p$. In our content-spreading model, the probability of "preserving" an edge between agents i and j corresponds to the probability of content spread between those two agents. However, in contrast to classical bond-percolation models, this probability is not fixed in our model. Instead, it depends on the opinions of agents i and j and on whether or not those two agents are adjacent to other agents that previously shared the content.

To analyze the structure of content spread in our model, we define an opinion-dynamics analog of the basic reproduction number from disease dynamics, and we use this *opinion reproduction number* and its associated critical value (which we call the *infodemic threshold*) to examine when an infodemic occurs in a network. We use generating functions to derive an analytical expression for the opinion reproduction number on configuration-model networks in the limit of infinitely many nodes. We then compare this analytical result with numerical simulations. We quantify the spread of content by calculating a variety of summary statistics—the total number of shared pieces of content, the longest adoption paths, the widths of dissemination trees, and structural virality—that were employed previously by other researchers.[2,75]

Our paper proceeds as follows. In Sec. II, we give a brief introduction to bounded-confidence models of opinion dynamics. In Sec. III, we describe a content-spreading process that draws inspiration from percolation theory and uses a bounded-confidence mechanism for spreading. In Sec. IV, we analyze the conditions that determine when we expect a large content-spreading cascade (i.e., an "infodemic"). To allow us to forecast whether or not an infodemic occurs in a network, we define an opinion-dynamics analog of the basic reproduction number from disease dynamics. In Sec. V, we quantify the total number of shared pieces of content on a finite-size network. In Sec. VI, we compute a variety of summary statistics to describe the spread of content. We illustrate how network structure and spreading-model parameters affect these statistics in configuration-model networks. In Sec. VII, we conclude and discuss our results. In the Appendix, Mason writes a few words about David Campbell and wishes him a wonderful 80th birthday.

II. BOUNDED-CONFIDENCE MODELS ON NETWORKS

Bounded-confidence models (BCMs)[34,35,37,76] of opinion dynamics include both consensus-seeking behavior and preferences for similar views (through "selective exposure"). BCMs have been used by many researchers to study social influence and group dynamics (including consensus, polarization, fragmentation, and other phenomena).[39]

We model a social network as a graph $G(V, \mathcal{E})$, where V (with $|V| = N$, so N is the "size" of the network) is the set of vertices (i.e., nodes) and $\mathcal{E} \subseteq V \times V$ is the set of edges. Each node represents an agent in some population, and each edge encodes a social and/or communication tie between two agents. We suppose that G is undirected and unweighted. Node i is *adjacent* to node j (and vice versa) if there is an edge between i and j. We assume that G is simple, so it has no self-edges or multi-edges. The graph G has an associated $N \times N$ *adjacency matrix* A, where $A_{ij} = 1$ if i is adjacent to j and $A_{ij} = 0$ otherwise. If desired, one can consider directed networks, weighted networks, and other generalizations of graphs. See Refs. 74, 77–79 for books and reviews about networks.

Suppose that each node i has a time-dependent opinion $x_i(t)$, which takes continuous values on a domain that we call the *opinion space*. For example, the opinion space can be the real line $\mathbb{R}$, a closed and bounded interval $[a, b] \subset \mathbb{R}$, or a subset of a higher-dimensional space $\mathbb{R}^k$. In a BCM, agents are receptive only to other agents that are within a distance c of their current opinion. More precisely, we say that agent i is *receptive* to agent j (and vice versa) if $d(x_i(t), x_j(t)) < c$, where d is a metric on the opinion space. The parameter c is called the *confidence bound*. Agents that are receptive to each other influence each other's opinions through an update rule. It is also possible to consider BCMs with asymmetric receptiveness, but we examine only symmetric situations.

III. A CONTENT-SPREADING PROCESS WITH A BOUNDED-CONFIDENCE MECHANISM

We study a content-spreading model that has a bounded-confidence mechanism for adopting an opinion and sharing content. However, in our model, receptive agents simply adopt the opinion that is espoused by content, rather than compromising their opinions as in a standard BCM.

The BCMs that we discussed in Sec. II describe the time-dependent opinions of the agents of a network. However, in real life, one usually does not have easy access to numerical values of people's opinions. One instead typically observes the content that is spread on social networks. Naturally, such content often reflects an underlying opinion or ideological belief. There is much empirical work on quantifying the dissemination and virality of content on social-media platforms like 𝕏 (i.e., the platform formerly known as Twitter),[11,12] and branching processes provide a natural model of content spread on social media. See, for example, Refs. 8, 10, 75, 80, and 81.

We seek to track the spread of fixed content with a fixed opinion x_0. In other words, the spreading process has no mutation of ideas or editorializing. In the context of social media, one can interpret the fixed-content assumption as accounts only sharing or reposting content, without any additions or changes. By contrast, real social-media systems have a rich ecosystem of content evolution.[82]

As we described in Sec. II, we model a social network as an undirected, unweighted, and simple graph $G(V, \mathcal{E})$ with $|V| = N$ nodes. The spread of content with opinion x_0 begins at a source node, which we select uniformly at random from V and label as node 0. We initialize all other nodes $i \in \{1, \ldots, N-1\}$ of a graph G with a state x_i that we draw from a distribution $\phi(x)$. In our analytical calculations, we assume either that there is a single source node or that the number of source nodes is much smaller than the network

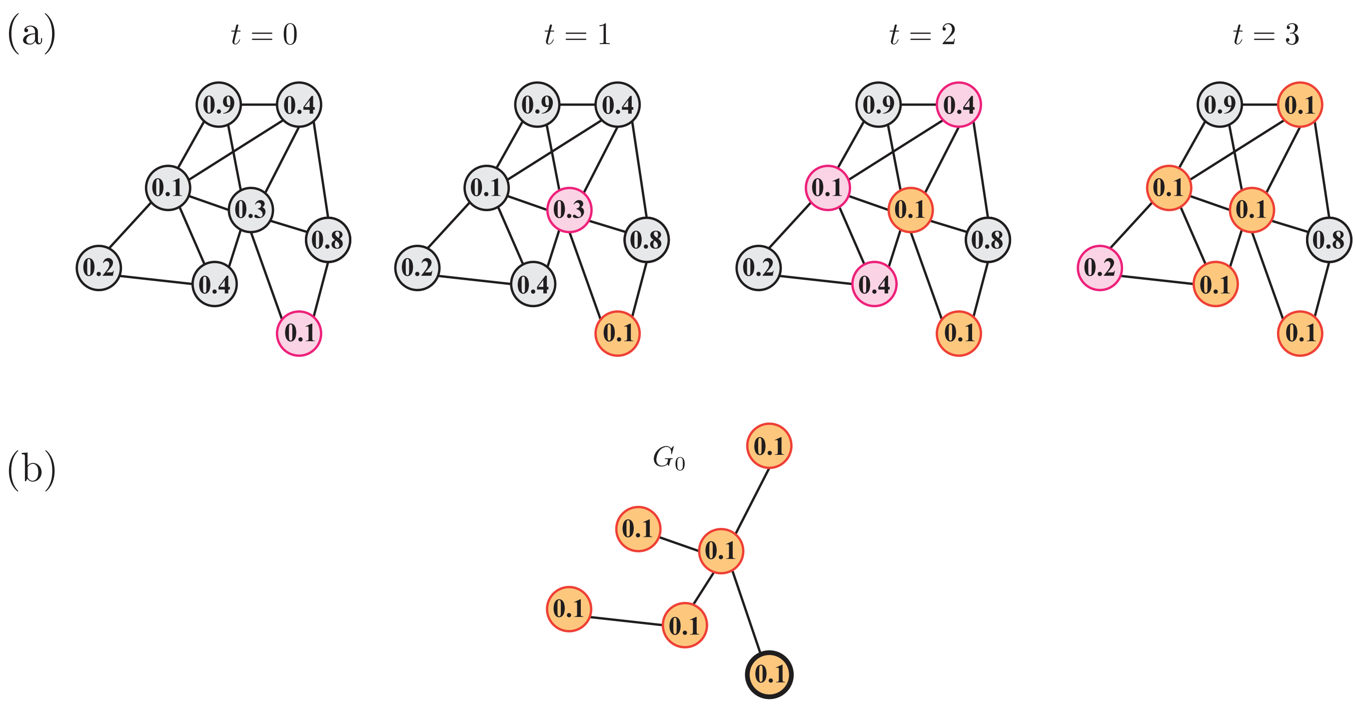

FIG. 1. A schematic illustration of our content-spreading process (see Algorithm 1). In panel (a), we show several consecutive update steps of Algorithm 1. We initialize a graph at time $t = 0$ with one active node (in pink), which we label as node 0. We suppose that the root node 0 has content state $x_0 = 0.1$. We initialize all other nodes j to have opinions $x_j \in (0, 1)$. In this example, the receptiveness parameter is $c = 0.35$. At each time step, any neighbors j of the previously active node that satisfy $|x_j - x_0| < c$ become active (i.e., turn pink). We thus add them to the dissemination tree G_0. In this example, Algorithm 1 terminates after four steps. In panel (b), we show the resulting dissemination tree G_0 in orange, with the root node outlined in black.

size N. We can then treat spreading processes that start from distinct nodes as independent of each other. We require $\phi(x)$ to be Riemann integrable.

Given an opinion space Ω, we refer to the opinion $x_0 \in \Omega$ of the content as the *content state*. We refer to the state $x_i \in \Omega$ of node $i \in \{1, \ldots, N-1\}$ as its *opinion*. In the present paper, we focus on $\Omega = [0, 1]$, although we show a situation with $\Omega = \mathbb{R}$ in Fig. 9. Let $c \in [0, 1]$ denote the *receptiveness parameter*, which is akin to the confidence bound of a typical BCM. In practice, we consider $c \in (0, 1/2]$, so agents are not receptive to agents with overly different opinions. The spread of content with opinion x_0 begins at node 0. The content spreads through a percolation-inspired process[53] via an update rule that is based on a bounded-confidence mechanism (see Algorithm 1). In Fig. 1, we show a schematic of our algorithm on a small network.

To study content spread on a network, we construct a dissemination tree G_0 with root node 0 to track the spreading process.[1,83] Content spreads as follows. Suppose that content state x_0 starts at node 0. We look at the set $\mathcal{N}_i$ of neighbors j of node i and select all nodes $j \in \mathcal{N}_i$ with an opinion x_j that satisfies $|x_j - x_0| < c$. These neighbors of i spread the content (i.e., they "activate" and change their opinions x_j to the content state x_0) because the distance between the opinion x_j and the content state x_0 is sufficiently small.

ALGORITHM 1. Our algorithm for the spread of content on a network.

Input: Model parameters c, x_0, $\phi(x)$, N and the degree distribution
Output: Dissemination tree G_0

Set `active nodes` $= 0$; `next nodes` $= \{\}$
while `active nodes` $\neq \{\}$ **do**
  for i in `active nodes` **do**
    `neighbors` $= \{j \mid A_{ij} = 1\}$
    for j in `neighbors` **do**

$$x_j = \begin{cases} x_i \text{ (add } i \text{ to next nodes)}, & \text{if } |x_j - x_0| < c \\ x_j, & \text{otherwise} \end{cases}$$

    end for
    for k in `next nodes` **do**
      $A_{ik} = 0$ for all i
    end for
  end for
  `active nodes` $=$ `next nodes`
  Add `next nodes` to G_0
  `next nodes` $= \{\}$
end while

We represent this activation by adding these nodes to the dissemination tree G_0 as child nodes of node i. We then repeat this process for each child node until there are no available neighbors that are left to activate. In other words, we continue this process until either all nodes are activated or until no active nodes have any remaining receptive neighbors.

In Algorithm 1, it is guaranteed that G_0 is a tree. If it is possible for a node to activate (which can occur only once), it activates the first time that it encounters the content of one of its neighbors. When two active nodes share a receptive neighbor, we choose the active node with the smaller index as the parent node of the newly activated neighbor. Modifications of this algorithm can result in a directed acyclic graph instead of a tree. It is worthwhile to examine such modifications in future work (e.g., if considering the effects of competing social contagions[84]).

IV. FORECASTING AN "INFODEMIC" ON A LARGE NETWORK

We now examine content spread on a network with infinitely many nodes. This situation approximates content spread on networks with many nodes (i.e., large N), particularly in the early stages of content spread.

We use analysis that was developed for models of infectious-disease spread on networks.[85–87] We use generating functions to obtain an expression for the expected number of neighbors of a node that spread its content. We refer to this expected number of neighbors as the *opinion reproduction number* R. The opinion reproduction number, whose critical value $R = 1$ is its associated *infodemic threshold*, is akin to the basic reproduction number R_0 in models of disease spread.[6,7]

A. Analysis

Suppose that content spreads on a configuration-model network[88] with degree distribution p_k. Let q_k denote the associated excess degree distribution. We generate each configuration-model network by uniformly randomly matching "stubs" (i.e., ends of edges) to each other. We remove all self-edges and multi-edges after matching stubs, so different networks can have slightly different degree sequences. The generating function g_0 of the degree distribution p_k and the generating function g_1 of the excess degree distribution q_k are

$$g_0(z) = \sum_{k=0}^{\infty} p_k z^k, \tag{1}$$

$$g_1(z) = \sum_{k=0}^{\infty} q_k z^k. \tag{2}$$

Given a node with k neighbors, we want to determine the probability that l of these neighbors spread a piece of content. To do this, we need to calculate the probability that content spreads along one edge. This single-edge transmission probability depends on the content state x_0, the receptiveness parameter c, and the distribution of the initial opinions x_i (with $i \in \{1, \ldots, N-1\}$).

Suppose that we draw the initial opinions x_i from a distribution with probability density function $\phi(x)$. The probability of content spread (i.e., transmission) along an edge depends on the probability of drawing a value x_i that lies within a distance c of x_0. The single-edge transmission probability is

$$s(x_0, c) = \int_{x_0-c}^{x_0+c} \phi(x)dx\,. \tag{3}$$

One common choice of the probability density function $\phi(x)$ comes from considering the uniform distribution on the interval $(0, 1)$. The probability of drawing an opinion x_i within a distance c of x_0 is then $2c$, unless x_0 is within c of either end of the boundary of $(0, 1)$. This yields

$$s(x_0, c) = \begin{cases} c + x_0\,, & x_0 \leq c \\ 2c\,, & x_0 \in (c, 1-c] \\ 1 + c - x_0\,, & x_0 > 1 - c\,. \end{cases} \tag{4}$$

Another common choice of $\phi(x)$ is the probability density function for a Gaussian distribution with mean μ and standard deviation σ. (In this case, the opinion space is $\mathbb{R}$.) This yields

$$s(x_0, c) = \frac{1}{2}\left[\operatorname{erf}\left(\frac{x_0 + c - \mu}{\sigma\sqrt{2}}\right) - \operatorname{erf}\left(\frac{x_0 - c - \mu}{\sigma\sqrt{2}}\right)\right], \tag{5}$$

where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ denotes the error function.

Using the single-edge transmission probability [see Eq. (3)], we see that the probability that l of k neighbors spread the content is

$$p(l|k) = \binom{k}{l}\left(s(x_0, c)\right)^l \left(1 - s(x_0, c)\right)^{k-l}. \tag{6}$$

Therefore, the probability generating function of the distribution of the number of content shares from a node to its neighbors is

$$\begin{aligned} \sum_{l=0}^{\infty}\sum_{k=l}^{\infty} p_k p(l|k) z^l &= \sum_{l=0}^{\infty}\sum_{k=l}^{\infty} p_k \binom{k}{l}\left(s(x_0, c)\right)^l \left(1 - s(x_0, c)\right)^{k-l} z^l \\ &= \sum_{k=0}^{\infty} p_k \sum_{l=0}^{k}\binom{k}{l}\left(s(x_0, c)\right)^l \left(1 - s(x_0, c)\right)^{k-l} z^l \\ &= \sum_{k=0}^{\infty} p_k \left(1 + (z-1)s(x_0, c)\right)^k \\ &= g_0(1 + (z-1)s(x_0, c))\,. \end{aligned} \tag{7}$$

To obtain the probability generating function of the distribution of the number of second neighbors (i.e., the neighbors of neighbors) of a node in a dissemination tree, we use a similar argument, but now we use the excess degree distribution instead of the degree distribution. This yields the probability generating function

$$g_1(1 + (z-1)s(x_0, c))\,. \tag{8}$$

Recall that content spread yields a directed acyclic graph (i.e., a dissemination tree) in our model. Additionally, consider a network G that is infinite (or at least large enough so that we can neglect finite-size effects). We then have that $g_a(1 + (z-1)s(x_0, c)) = g_1(1 + (z-1)s(x_0, c))$ is the probability generating function of the distribution of the number of nodes that spread content that they received directly from a single node at level a of a dissemination tree.

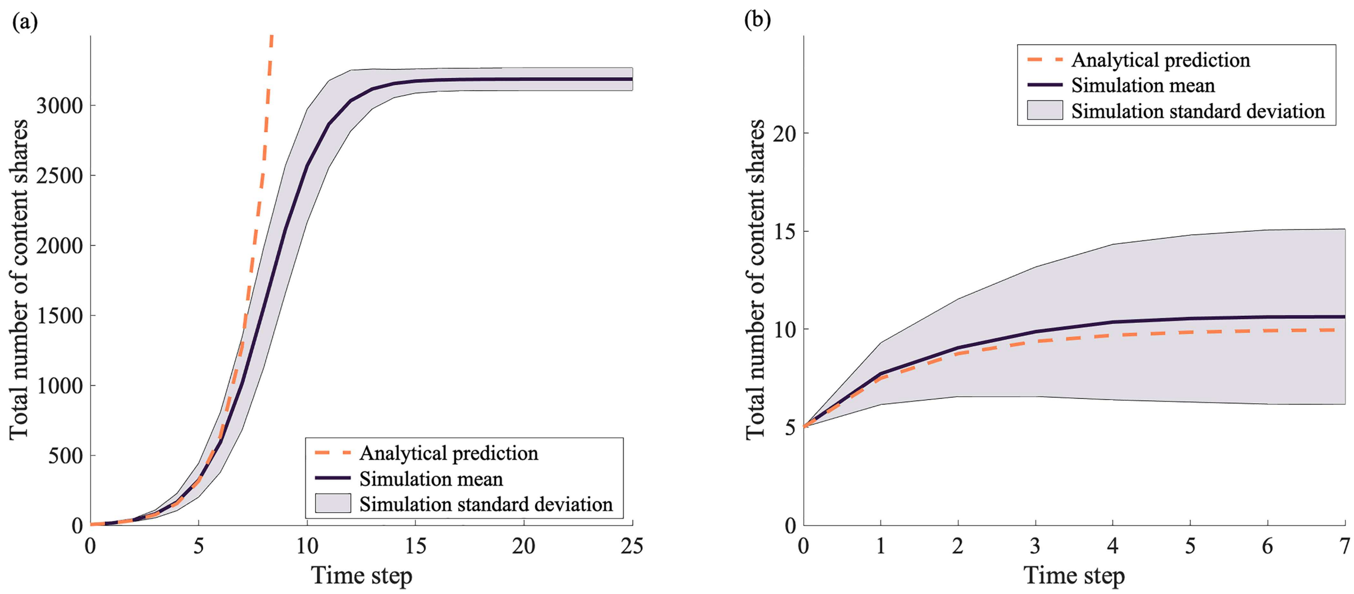

FIG. 2. A comparison of numerical simulations of our content-spreading model with our analytical predictions in (a) an infodemic situation (when the receptiveness parameter is $c = 0.2$, which implies that $R > 1$) and (b) a situation without an infodemic (when the receptiveness parameter is $c = 0.05$, which implies that $R < 1$). For each numerical simulation, we generate a 10 000-node configuration-model network with a degree sequence that we determine using a Poisson distribution with mean $\lambda = 5$. We draw the initial node opinions x_i uniformly at random from $(0, 1)$, and we set the content state to $x_0 = 0.5$. In each realization, we choose five source nodes uniformly at random to seed with content state x_0. In our analytical calculations, we assume that the content spread from each source node is independent, as the number of source nodes is much smaller than the total number of nodes. In each panel, the solid purple curve gives the total number of content shares (averaged over 100 realizations) as a function of time. In each realization, we draw a new degree sequence and a new set of node opinions. The light purple shaded region indicates the standard deviation for these 100 realizations. The dashed orange curve gives our analytical approximation.

Importantly, we are relying on the assumption that the number of times that a piece of content spreads is much smaller than the total number of nodes of a network.

In the early stages of content dissemination, the expected number of "grandchildren" (i.e., second neighbors) in a dissemination tree that starts from a node that spreads the content at level a is the opinion reproduction number

$$R = \frac{d}{dz} g_a(1 + (z-1)s(x_0, c))\bigg|_{z=1} = s(x_0, c) g_1'(1) = s(x_0, c)\frac{g_0''(1)}{g_0'(1)}, \qquad (9)$$

where the prime $'$ denotes differentiation with respect to z. Equation (9) relates the expected number of grandchildren to the expected number of children. When the number of content shares increases from one level to the next, the opinion reproduction number R (which is analogous to the basic reproduction number R_0 in models of infectious-disease spread[6]) is larger than 1. Similarly, a decrease in the number of content shares from one level to the next corresponds to $R < 1$. The critical value $R = 1$ is the *infodemic threshold*. This calculation is reminiscent of branching-process approaches that examine whether content spread tends to locally magnify or contract with time.[8] Given an opinion reproduction number R, we approximate the expected number of content

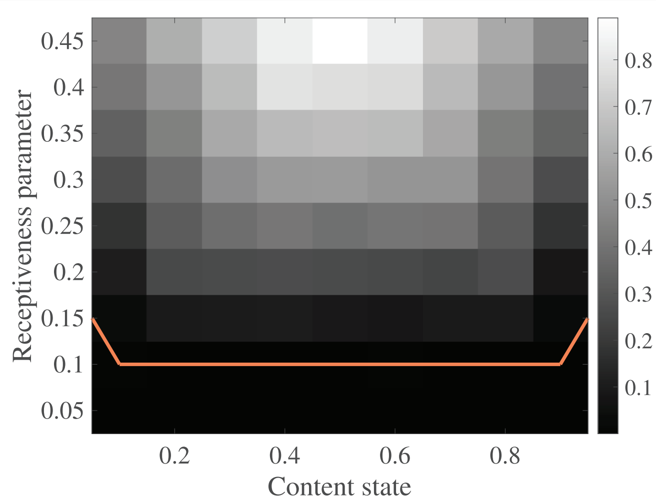

FIG. 3. The (x_0, c) phase diagram for 10 000-node configuration-model networks with degree sequences that we generate using a Poisson distribution with mean $\lambda = 5$. We vary both the content state x_0 and the receptiveness parameter c. The shading of each square indicates the mean, across 100 realizations, of the proportion of the nodes that share the content, with lighter shades indicating that more nodes share the content. In each realization, we choose five source nodes uniformly at random to seed with content state x_0. Additionally, in each realization, we draw a new degree sequence and a new set of node opinions. The orange solid curve shows the critical receptiveness value c^* for each x_0.

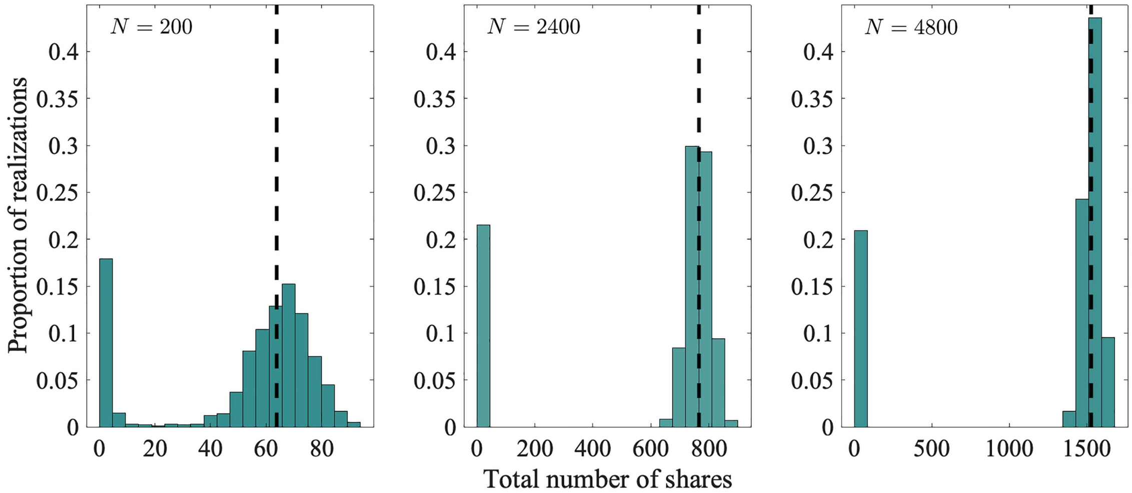

FIG. 4. Histograms of the total number of content shares in our content-spreading model across 1000 realizations for 200-node, 2400-node, and 4800-node configuration--model networks. The dashed vertical lines indicate the numbers of content shares (i.e., the dissemination-tree sizes) that we predict from our analysis. In each realization, we generate an *N*-node configuration-model network with a degree sequence that we obtain from a Poisson distribution with mean $\lambda = 5$. We draw the node opinions x_i uniformly at random from $(0, 1)$, and we set the content state to $x_0 = 0.5$ and the receptiveness parameter to $c = 0.2$. In each realization, we draw a new degree sequence and a new set of node opinions.

shares up to and including time step T as $n_0 \sum_{t=0}^{T} R^t$, where n_0 is the number of source nodes and we treat content that spreads from different source nodes as independent. Given our assumptions, we expect our approximation to be reasonable in the first few generations (i.e., levels) of a dissemination tree. These initial generations describe the early stages of content spread.

We now calculate R explicitly for an example in which we draw the initial node opinions uniformly at random from the interval $(0, 1)$. In a configuration-model network with a Poisson degree distribution with mean λ, the opinion reproduction number is

$$R = s(x_0, c)\frac{\lambda^2}{\lambda} = s(x_0, c)\lambda \tag{10}$$

because $g_0(z) = e^{-\lambda(1-z)}$. Substituting Eq. (4) into Eq. (10) allows us to calculate the opinion reproduction number in terms of the content state x_0, the receptiveness parameter c, and λ. At the infodemic threshold $R = 1$, the critical value c^* of the receptiveness parameter that determines whether content spread increases locally or decreases locally is

$$c^* = \begin{cases} \frac{1}{\lambda} + x_0\,, & x_0 \le c^* \\ \frac{1}{2\lambda}\,, & c^* < x_0 \le 1 - c^* \\ \frac{1}{\lambda} - 1 + x_0\,, & x_0 > 1 - c^*. \end{cases}$$

When the receptiveness parameter $c > c^*$, the expected number of content shares increases from one level to the next in the early stages of content spread, so we expect the content to take hold in a network. Unsurprisingly, it is easiest to exceed the infodemic threshold $R = 1$ for large values of the receptiveness parameter c and for networks with large expected mean degree λ. The content state x_0 has less impact than the receptiveness parameter and the expected mean degree, except when the content state is near 0 or 1.

B. Simulations

We now compare our analytical results from Sec. IV A to numerical simulations of our content-spreading model. We simulate the model on 10 000-node configuration-model networks with degree sequences that we determine using a Poisson distribution with mean $\lambda = 5$. We draw the initial opinions uniformly at random from the interval $(0, 1)$. In Fig. 2(a), we show a scenario in which our analysis indicates that we are above the infodemic threshold (i.e, $R > 1$). In Fig. 2(b), we show a scenario in which our analysis indicates that we are below the infodemic threshold (i.e., $R < 1$). In this computation, the mean total number of content shares is very small. (It is about 10.)

In Fig. 3, we show a phase diagram that summarizes when our content-spreading model experiences an infodemic for different values of the content state x_0 and receptiveness parameter c. In orange, we show the critical value c^* that we obtain analytically using the infodemic threshold $R = 1$.

V. QUANTIFYING CONTENT SPREAD

Now that we have analyzed the onset of infodemics in our content-spreading model on configuration-model networks, we measure the sizes (i.e., the numbers of nodes) in the resulting dissemination trees. The size of a dissemination tree equals the total number of content shares. Studying dissemination trees allows us to explore the long-term behavior of our content-spreading model. We thereby complement our examination of early-stage spreading (see Sec. IV).

A. Analysis of the total number of content shares

Given the number of nodes and the degree distribution of a configuration-model graph G, a content state x_0, a receptiveness

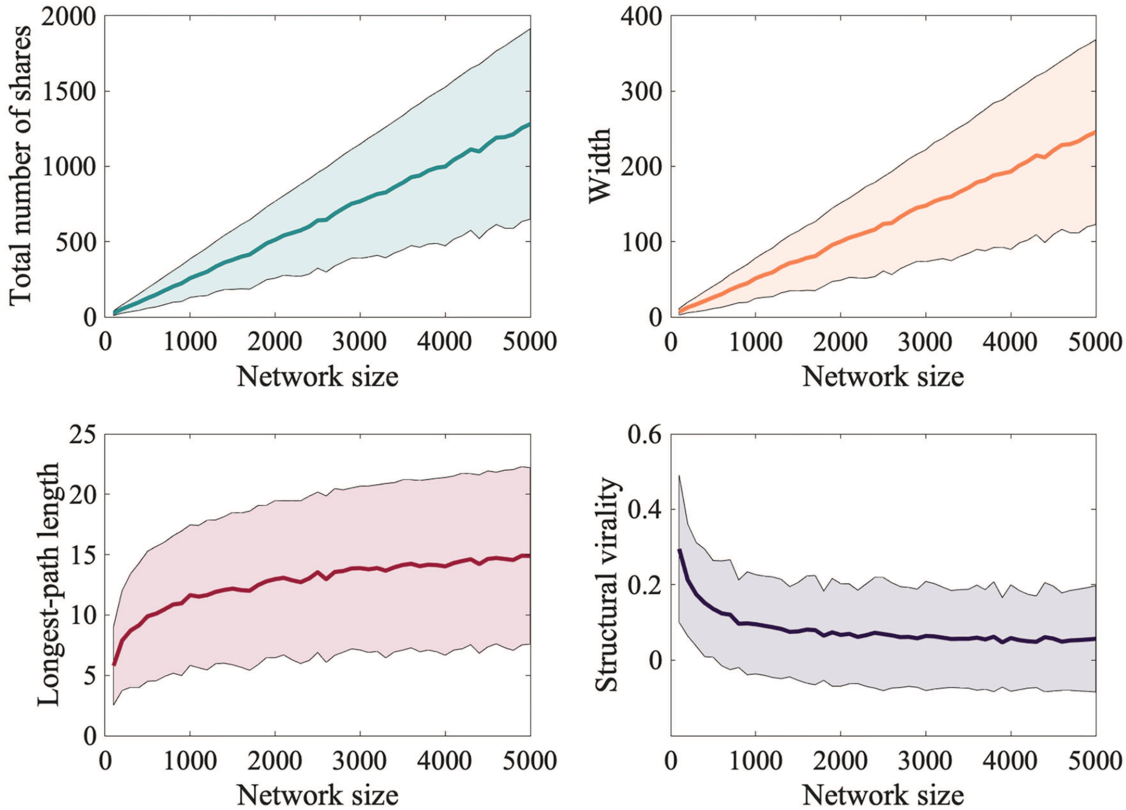

FIG. 5. The effect of varying the network size N on the total number of content shares, the width, the longest-path length, and the structural virality of dissemination trees of our content-spreading model on configuration-model networks. The solid curves give means across 1000 realizations, and the shaded regions give the standard deviations. In each realization, we generate an N-node configuration-model network with a degree sequence from a Poisson distribution with mean $\lambda = 5$. We vary N from 100 to 5000 in increments of 100. Each realization has different initial node opinions, which we draw uniformly at random from $(0, 1)$. The content state is $x_0 = 0.5$ and the receptiveness parameter is $c = 0.2$. In each realization, we draw a new degree sequence and a new set of node opinions. Both the total number of content shares (i.e., the dissemination-tree size) and the width grow linearly with N. The longest-path length grows quickly at first as we increase N, and then it grows much more slowly with N. The structural virality appears to saturate at a constant value for sufficiently large N.

parameter c, and a distribution $\phi(x)$ of opinions, we again use generating functions (as in Sec. IV A) to estimate the total number of content shares (i.e., the size) of a dissemination tree G_0. This analysis treats content spread as a percolation process. See Ref. 74 for a detailed description of this approach.

Let the "spreading set" $\mathcal{S}$ of a network denote the set of nodes that have shared a piece of content. The spreading set $\mathcal{S}$ has SN elements (i.e., there are a total of SN content shares), where S is the fraction of a network's nodes that have shared that content. Node opinions are independent of network connectivity in our content-spreading model, so the probability that a node is in $\mathcal{S}$ equals the probability that it is receptive to the content multiplied by the probability that it belongs to the connected component of a source node. In this calculation, we assume that there is a single source node. Therefore, SN is the size of the connected component of one source node. Let u denote the mean probability of not being connected to this component via a particular neighbor. We then have

$$S = \left(\int_{x_0-c}^{x_0+c} \phi(x)\, dx\right)\left(1 - \sum_{k=0}^{\infty} p_k u^k\right) = s(x_0, c)\left(1 - g_0(u)\right), \tag{11}$$

where we recall that p_k is the probability that a node has degree k and $g_0(u)$ is the generating function of the degree distribution.

To calculate S, we need to determine u. A particular node i is not connected to the spreading set $\mathcal{S}$ via a particular node j either because it is not receptive to the content or because it is not in the same component as the content. In the latter case, for a connected network G, node i is isolated from the content spread because of non-receptive nodes. For a node j with k edges aside from the one to node i, the probability that one of these scenarios occurs is

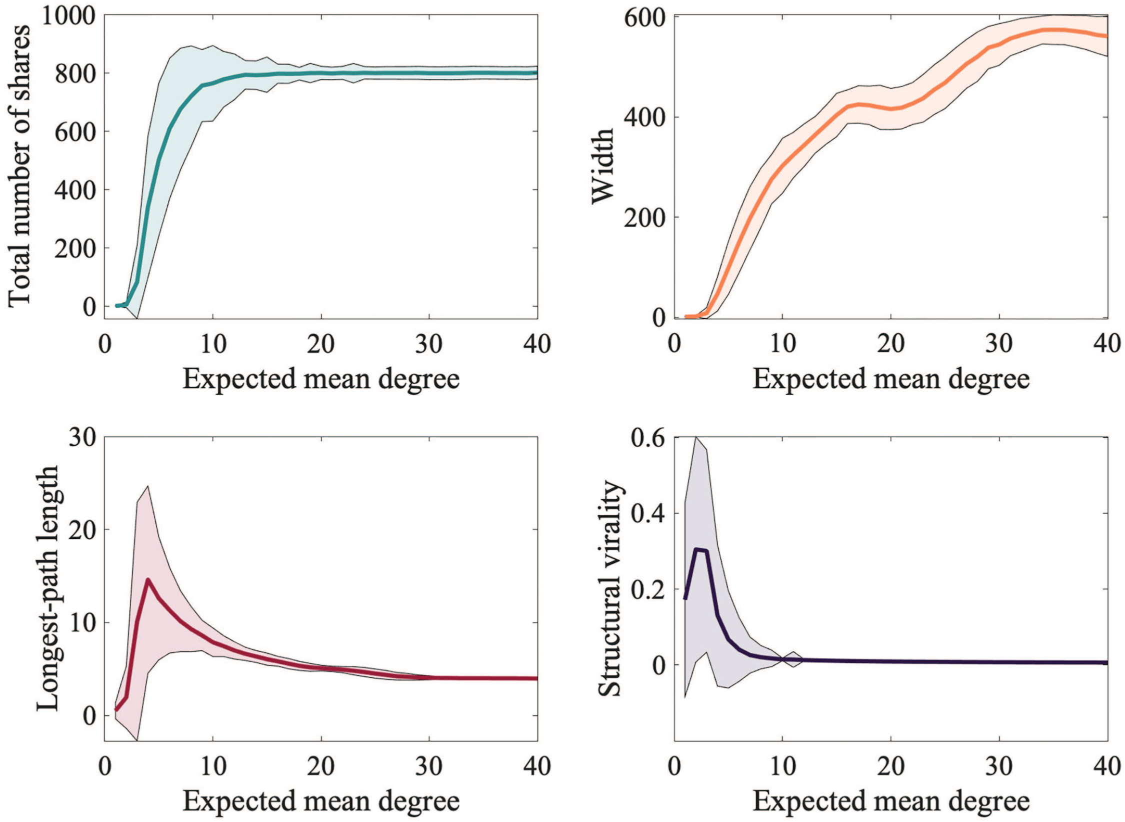

FIG. 6. The effect of varying the expected mean degree of a network on the total number of content shares, the width, the longest-path length, and the structural virality of dissemination trees of our content-spreading model on configuration-model networks. The solid curves give means across 1000 realizations, and the shaded regions give the standard deviations. In each realization, we generate a 2000-node configuration-model network with a degree sequence from a Poisson distribution with mean λ. We vary λ from 1 to 40 in increments of 1. Each realization has different initial node opinions, which we draw uniformly at random from (0, 1). The content state is $x_0 = 0.5$ and the receptiveness parameter is $c = 0.2$. In each realization, we draw a new degree sequence and a new set of node opinions. The total number of content shares grows initially and then saturates at about 800. The width also tends to increase with λ, with a possible plateau in a small interval near $\lambda = 20$. The longest-path length and structural virality increase at first and then decrease, eventually leveling off at a constant value for sufficiently large expected mean degrees. In our simulations, the longest-path length has a maximum at about $\lambda = 4$ and the structural virality has a maximum at about $\lambda = 3$.

$1 - s(x_0, c) + s(x_0, c)u^k$. Therefore,

$$u = \sum_{k=0}^{\infty} q_k \left(1 - s(x_0, c) + s(x_0, c)u^k\right) = 1 - s(x_0, c) + s(x_0, c)g_1(u)\,, \qquad (12)$$

where q_k again denotes the probability of having k neighbors other than the spreading edge (i.e., the excess degree is k) and $g_1(u)$ again denotes the generating function of the excess degree distribution. By inspection, $u = 1$ is always a solution of Eq. (12); this solution entails not having an infodemic. We are interested in whether or not there are also solutions $u \in (0, 1)$.

In some special cases, it is possible to solve for u analytically, but it is typically difficult (or even impossible) to do so even when $g_1(u)$ has a simple closed-form expression. However, one can find a root $u^* \in (0, 1)$ of Eq. (12) numerically using either an explicit expression for $g_1(u)$ or an approximation of it.[74] One then substitutes u^* into Eq. (11) to obtain an approximation of S. A solution $u^* \in (0, 1)$ yields an approximation of S when there is an infodemic.

In Fig. 4, we compare our approximation of the total number of content shares (SN) to the number of content shares in numerical simulations of our model on configuration-model networks of three different sizes. For each size (200, 2400, and 4800 nodes), we generate 1000 configuration-model networks with degree sequences from a Poisson distribution with mean $\lambda = 5$. We plot histograms of the total numbers of content shares, and we observe good agreement between our analytical and numerical results.

VI. SUMMARY STATISTICS FOR DISSEMINATION TREES

In this section, we perform numerical experiments and study the output of our bounded-confidence content-spreading model on

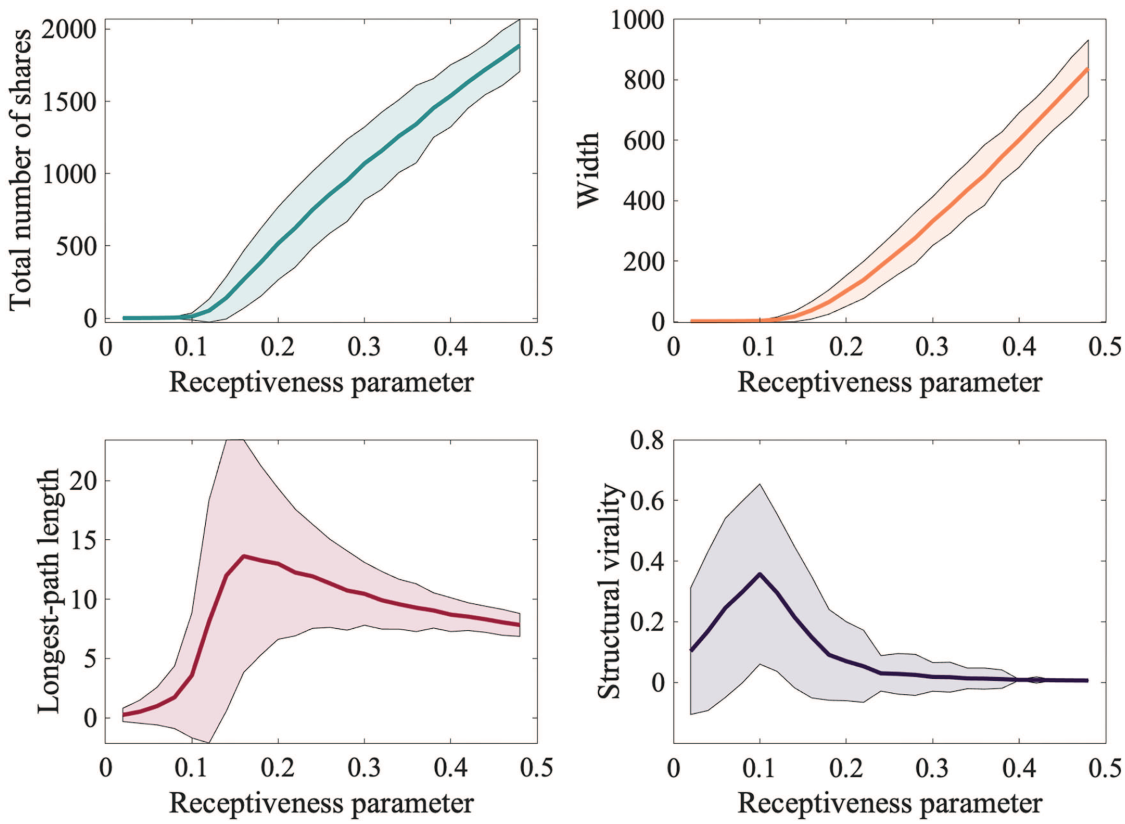

FIG. 7. The effect of varying the receptiveness parameter c on the total number of content shares, the width, the longest-path length, and the structural virality of dissemination trees of our content-spreading model on configuration-model networks. The solid curves give means across 1000 realizations, and the shaded regions give the standard deviations. In each realization, we generate a 2000-node configuration-model network with a degree sequence from a Poisson distribution with mean $\lambda = 5$. Each realization has different initial node opinions, which we draw uniformly at random from $(0, 1)$. The content state is $x_0 = 0.5$. We vary c from 0.02 to 0.48 in increments of 0.02. In each realization, we draw a new degree sequence and a new set of node opinions. The total number of content shares and the width are very small until about $c = 0.1$, and then they grow with c. Both growth rates seem roughly linear after some earlier slow growth, although the curve for the total number of content shares seems to be concave down. The longest-path length increases initially with c before reaching a maximum and then decaying. The structural virality also appears to increase initially with c before reaching a maximum and then decaying to a constant value. The standard deviations of the longest-path length and the structural virality are large.

configuration-model networks. In each realization of these numerical simulations, we activate a single source node. We quantify the features of our model by computing four summary statistics to describe the dissemination trees that we obtain in our simulations.

- **Total number of content shares.** One way to measure the effectiveness of content spread is to count the total number of nodes that spread (i.e., "adopt") the content. This quantity, which we studied in Sec. V, is equal to the number of nodes in the dissemination tree G_0.
- **Length of a longest adoption path.** An adoption path is a path in a dissemination tree from a source node to another node in the tree.[83] We measure the "depth" of content spread by calculating the length of a longest adoption path.
- **Width.** One can arrange a dissemination tree G_0 with the source node at the top (i.e., at level 0), nodes with adoption-path length 1 at level 1, nodes with adoption-path length 2 at level 2, and so on. The width of a dissemination tree is the largest number of nodes that adopt the content in a single level. Mathematically, the width is

$$\max_{b \in \{1,\ldots,l_*\}} (\text{number of nodes with adoption-path length } b) ,$$

where l_* is the length of a longest adoption path.
- **Structural virality.** Structural virality (i.e., the Wiener index) was used by Goel *et al.*[1] to quantify the viral nature of content spread. The structural virality v is the mean shortest-path length between nodes in a dissemination tree G_0. Mathematically, the structural virality is

$$v = \frac{1}{N(N-1)} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} d_{ij} ,$$

where d_{ij} is the length of a shortest path between nodes i and j.

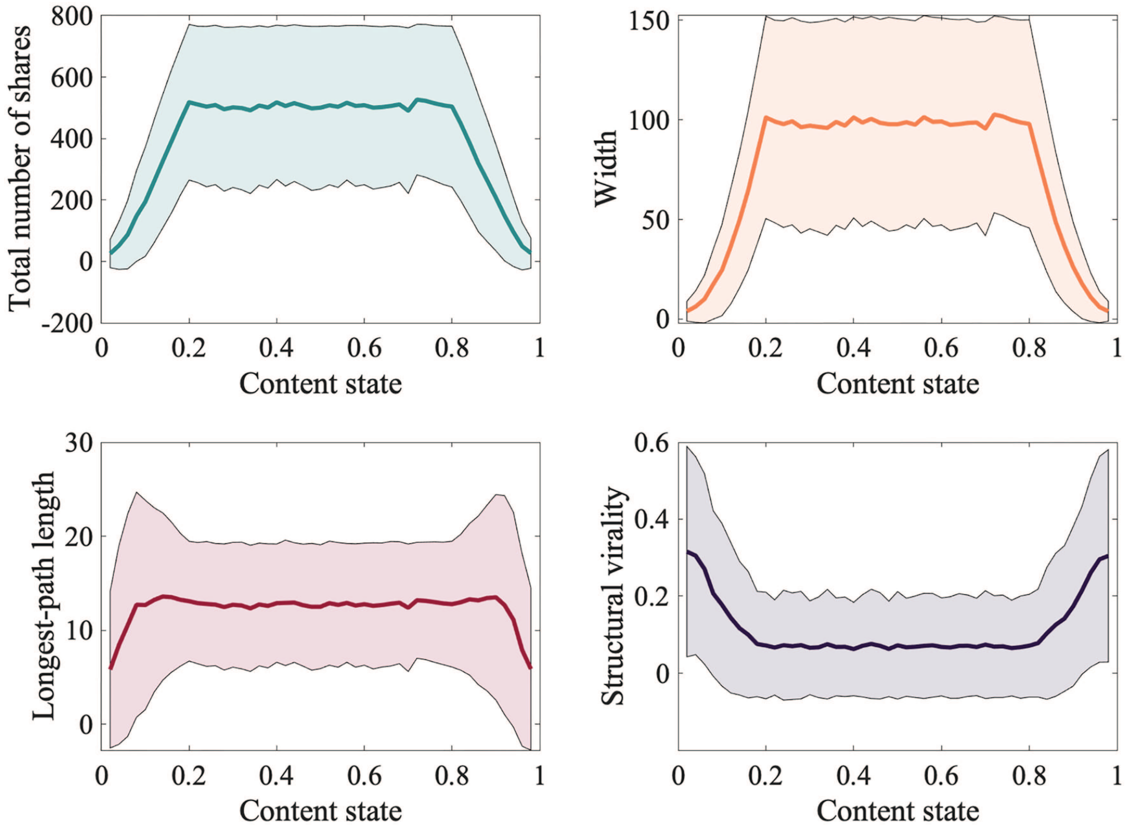

FIG. 8. The effect of varying the content state x_0 on the total number of content shares, the width, the longest-path length, and the structural virality of dissemination trees of our content-spreading model on configuration-model networks with initial node opinions from a uniform distribution. The solid curves show means across 1000 realizations, and the shaded regions give the standard deviations. In each realization, we generate a 2000-node configuration-model network with a degree sequence from a Poisson distribution with mean $\lambda = 5$. Each realization has different initial node opinions, which we draw uniformly at random from $(0, 1)$. The receptiveness parameter is $c = 0.2$. We vary x_0 from 0.02 to 0.98 in increments of 0.02. In each realization, we draw a new degree sequence and a new set of node opinions. When $x_0 \in (0.2, 0.8)$, the total number of content shares, the width, the longest-path length, and the structural virality are all roughly constant. This is not surprising because we draw initial node opinions uniformly at random, so the transmission probability for content to spread along an edge depends only on c in this interval [see Eq. (4)]. The symmetry of these summary statistics with respect to x_0 is also clear from Eq. (4). Content states that are closer to the boundary (i.e., closer to either 0 or 1) of opinion space produce dissemination trees with fewer total content shares, smaller widths, and shorter longest paths. However, the structural viralities of such "extreme" content states are slightly larger.

One natural question to ask is how the number of nodes of a network affects the spread of content on that network. We study this question by increasing the network size N for fixed expected mean degree λ (see Fig. 5), where we again consider a Poisson degree distribution with mean λ. With this construction, the expected number of edges remains constant as we increase N. As implied by our analysis in Sec. V, the total number of content shares grows linearly with N [with a slope indicated by Eq. (11)]. The dissemination-tree width also appears to grow linearly with N. The longest-path length and structural virality change more dramatically for small N than for large N. The former appears to grow very slowly for large N, and the latter appears to eventually saturate at a constant value.

In Fig. 6, we examine the impact of increasing the expected mean degree λ on the dissemination-tree statistics. The total number of content shares increases with λ before saturating at a constant value. The width also tends to increase with λ, with a possible plateau in a small interval near $\lambda = 20$. The longest-path length and structural virality have maxima for small values of λ. When λ is large, it is seemingly possible for the content to be adopted by all receptive nodes that are in the connected component of a source node in just a few time steps, yielding a dissemination tree with a large width, short adoption paths, and a small structural virality. The maxima in the longest-path length and structural virality indicate that a spreading process persists longer (and hence travels farther from a source node) when nodes have fewer neighbors on average, provided the mean degree is large enough for an infodemic to occur.

Varying the receptiveness parameter c (see Fig. 7) yields similar dissemination-tree statistics as varying the expected mean degree. We again observe maxima in the longest-path length and structural virality. These maxima likely arise because larger receptiveness values favoring faster content spread, although the longest-path length

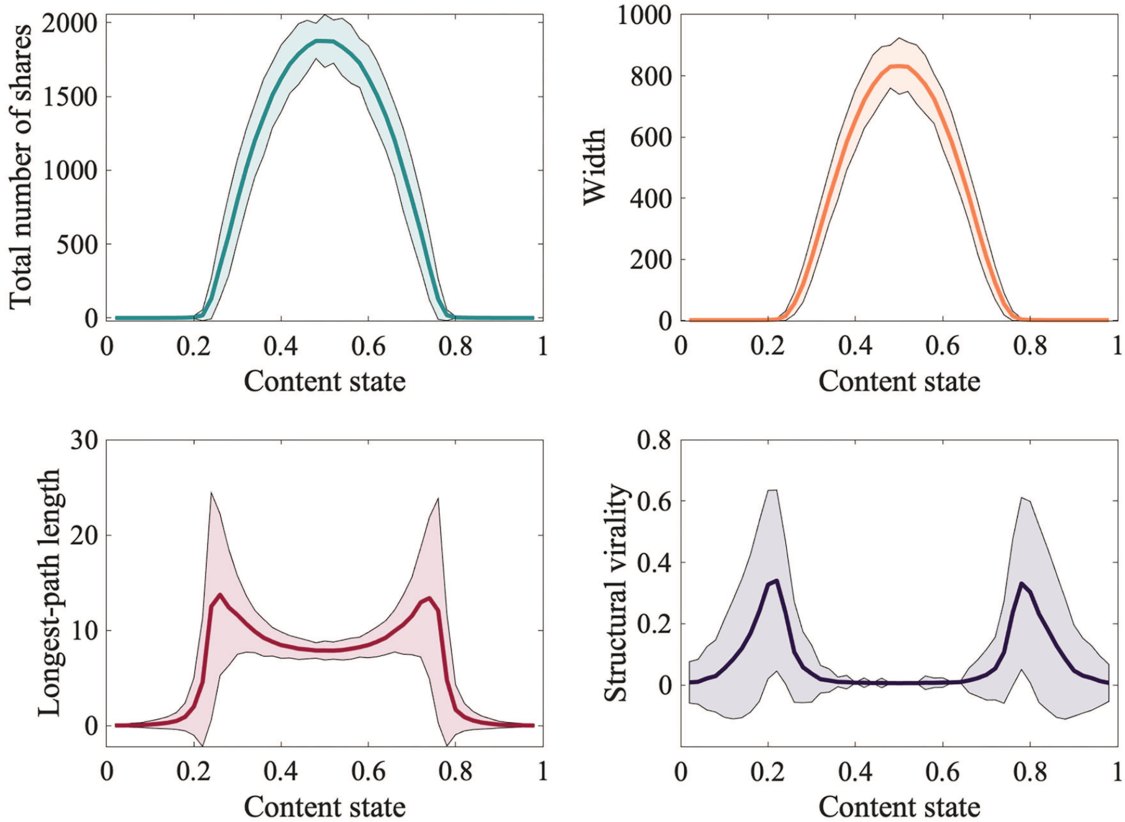

FIG. 9. The effect of varying the content state x_0 on the total number of content shares, the width, the longest-path length, and the structural virality of dissemination trees of our content-spreading model on configuration-model networks with initial node opinions that we draw from a Gaussian distribution with mean 0.5 and standard deviation 0.1. In this figure, the opinion space is $\mathbb{R}$. This opinion space and the initial opinion distribution differ from the ones in Fig. 8. The solid curves give means across 1000 realizations, and the shaded regions give the standard deviations. In each realization, we generate a 2000-node configuration-model network with a degree sequence from a Poisson distribution with mean $\lambda = 5$. The receptiveness parameter is $c = 0.2$. We vary the content state x_0 from 0.02 to 0.98 in increments of 0.02. In each realization, we draw a new degree sequence and a new set of node opinions.

and structural virality both have large standard deviations in this experiment. We observe evidence of a phase transition in the total number of content shares at the value of the receptiveness parameter c that we predicted in our analysis in Sec. IV A. There also appears to be an accompanying phase transition in the dissemination-tree width. In these simulations, the content state is $x_0 = 0.5$. We observe qualitatively similar behavior with content states (e.g., $x_0 = 0.15$) that are closer to the boundary (i.e., either 0 or 1) of opinion space but still in the infodemic regime. In our repository (see the folder "supplementary-figures" of Ref. 89), we include analogous figures to Fig. 6 and Fig. 7 for such content states.

We also examine the impact of the content state on our dissemination-tree statistics for two different initial opinion distributions [see Eq. (3)]. In Fig. 8, we show results of simulations in which we draw the initial node opinions from a uniform distribution on the interval $(0, 1)$. In Fig. 9, we show results of simulations in which we draw the initial node opinions from $\mathbb{R}$ using a Gaussian distribution with mean 0.5 and standard deviation 0.1. As expected, we obtain smaller values of all four summary statistics when the content state lies in the tails of the Gaussian distribution (and when it lies near either end of the boundary of the interval $(0, 1)$ for the uniform distribution). We observe maxima in the total number of content shares and dissemination-tree width at the distribution mean (and also for a very large interval around it for the uniform distribution). For the Gaussian distribution, the content states that yield the largest longest-path lengths and structural viralities occur away from the mean (and are symmetric), indicating that content can spread farther from a source node in these situations than when the content state equals the mean initial opinion. These dissemination trees are longer and narrower than those that we obtain when the content state equals the mean initial opinion. However, if the content state is too extreme, then it does not spread very much.

VII. CONCLUSIONS AND DISCUSSION

We examined "infodemics," in the form of large content-spreading cascades, in a bounded-confidence content-spreading model on configuration-model networks. To do this, we defined an "opinion reproduction number" that is analogous to the basic reproduction number of disease dynamics. By examining whether the opinion of the content and the receptiveness of individuals yield an opinion reproduction number that is larger than 1, we investigated when content propagates widely in a network of agents and when it does not.

We quantified the size and structure of content spread by measuring several properties—tree size, tree width, the length of the longest adoption path(s), and structural virality—of dissemination trees, and we thereby illustrated how network structure and spreading-model parameters affect the spread of content. We found that larger networks, larger expected mean degrees, larger receptiveness, and content states near the mean of an opinion distribution all promote larger total numbers of content shares, including the possibility of sharing the content to many new nodes in a single time step. Additionally, when the expected mean degree and receptiveness are small but in the regime where the spreading process is above the infodemic threshold, there can be longer dissemination trees than when the expected mean degree or receptiveness are larger, even though the total number of content shares is smaller. This indicates that content is spreading farther but less widely from a source agent (i.e., an agent that originally posts a piece of content).

There are many interesting ways to extend our work. A particularly relevant extension is to examine the effects of purposeful choices of source nodes, such as to try to promote influence maximization.[72,90] (An important point for such efforts is that our model takes a perspective that suggests novel generalizations of independent-cascade models.) One can also examine the effects of competing social contagions,[81] which perhaps are spread by agents with different political perspectives or agendas. Another worthwhile future direction is to adapt the recently developed "distributed reproduction numbers"[91] from disease spread to content spread. It is also desirable to extend our study of content-spreading dynamics to other network structures. One key direction is to consider more complicated network models (such as generalizations of configuration models that incorporate various types of heterogeneities[92]) and real-world networks. It is also natural to extend our investigation to more complicated types of networks, such as multilayer networks (which allow multiple types of social connections and communication channels),[93] hypergraphs (which allow simultaneous interactions between three or more agents),[94] and adaptive networks (e.g., to incorporate relationship changes, such as "unfollowing" or "unfriending" on social-media platforms).[95]

Other viable extensions of our model directly exploit the link that it establishes between opinion dynamics and percolation processes. It seems particularly exciting to study scenarios with rich interplays between the opinion of content and the opinions of agents. For example, one can incorporate mutations of content opinions[13,82] through averaging the opinion of content with the opinions of the agents that spread it. A related extension entails allowing agent opinions to evolve with time through their interactions with each other (as in classical bounded-confidence models, such as the Deffuant–Weisbuch model[96]) while content with a fixed opinion spreads on a network. It will be fascinating to explore how these extensions affect the properties of infodemic thresholds and spreading patterns.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Heather Z. Brooks: Conceptualization (lead); Formal analysis (lead); Investigation (lead); Methodology (lead); Funding acquisition (equal); Project administration (equal); Software (lead); Visualization (lead); Writing – original draft (lead); Writing – review & editing (equal). **Mason A. Porter:** Conceptualization (supporting); Funding acquisition (equal); Project administration (equal); Supervision (lead); Writing – review & editing (equal).

DATA AVAILABILITY

All data in this paper are the output of numerical computations. We performed computations and constructed visualizations using MATLAB. We have created a public repository that provides the code to reproduce our numerical simulations. It is available at Ref. 89.

APPENDIX: A FEW WORDS ABOUT DAVID CAMPBELL (BY MASON A. PORTER)

It is a great honor to contribute to *Chaos*'s special issue in celebration of David Campbell's 80th birthday. David is one of my favorite people in science, and I deeply appreciate being asked to contribute an article to this Festschrift. David Campbell's research contributions and, especially, his foundational role in *Chaos* are well known. I could write about that, but I will leave that to others. What is especially important is that David is a great human being. He has always been fair to me and he has been very good to me, as he has been to so many others.

I first interacted with David when I was a postdoc at Georgia Tech. I was naïvely trying to publish a modified version of my doctoral thesis[97] as a review article, and I submitted it to the journal *Physics Reports*. David, who I had not yet met in person, was the handling editor for my submission. My paper was rejected, but what stood out in that experience were the care and decency with which David handled things. All that I want with a manuscript submission—and all that anyone should ever want with a manuscript submission, as both a necessary and sufficient condition—is to be treated fairly and with dignity. David saw a teaching opportunity, and he could see that I had things to learn. He went beyond the call of duty and even asked the referee to disclose his identity to help with that teaching. By contrast, as we all see repeatedly, many journal editors seem to be bean counters, rather than actual editors. The rejection of my paper was not my ideal outcome, but it was the correct outcome, and I learned a lot from the experience. In my own editing roles, I have tried to draw on the approach and lessons that David exemplified in this story.

David continued looking out for me, as he has looked out for many others. Having—I think?—still not yet met in person, David invited me to give a talk as part of a special session at the 2005 American Physical Society (APS) March Meeting about the celebrated Fermi–Pasta–Ulam–Tsingou (FPUT) problem and to contribute an article about the dynamics of Bose–Einstein condensates[98] to a special issue of *Chaos* about the FPUT problem. David's talk had a pop-music reference (which was about the Eagles, if memory serves), and he mentioned that he figured that I would appreciate the musical reference. (He was right.) Four of us who spoke in the special session coauthored a popular article about the FPUT problem.[99] I eventually wrote a variety of papers on the FPUT problem (and on related problems), including one[100] that turned out to be refereed by David and one of his students. It was a particularly fair and thorough referee report—it was the type of report that scientists dream of getting and was a welcome reminder that sometimes the reviewing process actually works how it is supposed to—that my coauthors and I were working hard to address. I remember e-mailing David during our revision process to tell him about our paper, which concerns heterogeneities in FPUT lattices, because I figured that he would be interested it. My e-mail to David either was about something else entirely or was because I had just seen a paper of his on a related topic. Without any notion that David was our referee, I told him in my e-mail about our paper and that we were currently dealing with a tough-but-fair referee report. I found out from David's response that he and his student had written that report. Once again, David was doing things in a way that helps authors while upholding rigorous standards. In other words, David once again did things in the way that they are supposed to be done.

A recurring theme in my stories is David looking out for people (especially junior scientists), treating people fairly while simultaneously ensuring rigorous standards, and doing things the right way in scientific editing and reviewing. We all have peer-review horror stories, and publishing can cause many frustrations. However, some scientists consistently do things the right way, and David is one of them.

The best compliment that somebody like me who comes from Jewish heritage can pay to another person is to call them a "mensch" (which, essentially, is saying that that person is a decent human being, as they are a person of integrity and honor). David Campbell is a mensch.

Happy birthday, David!

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