

Oscillatory and excitable dynamics in an opinion model with group opinions

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In traditional models of opinion dynamics, each agent in a network has an opinion and changes in opinions arise from pairwise (i.e., dyadic) interactions between agents. However, in many situations, groups of individuals possess a collective opinion that can differ from the opinions of their constituent individuals. In this paper, we study the effects of group opinions on opinion dynamics. We formulate a hypergraph model in which both individual agents and groups of three agents have opinions, and we examine how opinions evolve through both dyadic interactions and group memberships. We find for some parameter values that the presence of group opinions can lead to oscillatory and excitable opinion dynamics. In the oscillatory regime, the mean opinion of the agents in a network has self-sustained oscillations. In the excitable regime, finite-size effects create large but short-lived opinion swings (as in social fads). We develop a mean-field approximation of our model and obtain good agreement with direct numerical simulations. We also show—both numerically and via our mean-field description—that oscillatory dynamics occur only when the numbers of dyadic and polyadic interactions of the agents are not completely correlated. Our results illustrate how polyadic structures, such as groups of agents, can have important effects on collective opinion dynamics.

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I. INTRODUCTION

The opinions of individuals in a social network often change when they are exposed to the opinions and actions of other individuals. The ensuing opinion dynamics of such individuals (i.e., “agents”) has received considerable attention from sociologists [1], economists [2], political scientists [3,4], applied mathematicians, theoretical physicists [3,5–8], and many others. Researchers have studied models of opinion dynamics on social networks to gain insight into phenomena such as the propagation of false or misleading information [9,10], the emergence of consensus opinions [7,11,12], and the formation of echo chambers [13,14]. See Refs. [15,16] for reviews of opinion models.

Models of opinion dynamics necessarily involve many assumptions about the nature of the opinions of individuals, the interactions between individuals, and how such interactions affect the opinions of other individuals [17]. Most opinion models assume that agent opinions change as a result of pairwise (i.e., dyadic) interactions. In opinion models, agent opinions are typically either real-valued scalars or real-valued vectors (e.g., if one wants to simultaneously model opinions of multiple things). In some models, the opinions have discrete values; in others, they have continuous values, such as in an interval of the real line [16]. Researchers typically consider discrete-value opinions when examining phenomena in which individuals or other entities make discrete choices, such as when people vote for a candidate for a political office. By contrast, researchers often consider continuous-valued opinions when they want to explicitly account for a wide spectrum of

views, such as political outlooks that range from very liberal to very conservative. There are numerous opinion models [16,18], which researchers study on networks to examine how social structures affect opinion dynamics. Examples of opinion models include the DeGroot consensus model [19], voter models [20,21] and their generalizations [22], majority-rule models [23], and bounded-confidence models [24].

A basic assumption in most opinion models is that only the individual agents, which are represented by the nodes of a network, are endowed with opinions. In the present paper, we relax this assumption by allowing groups of nodes to hold collective opinions. Social groups, which range in scope from family and friendship units to large political and corporate organizations, help shape the fabric of society [25–27]. In many situations, it is reasonable to posit that social groups themselves can possess opinions. For example, large corporate organizations sometimes take public stances on social issues [28]. Additionally, courts such as the United States Supreme Court hear cases and document collective opinions through their decisions on these cases [29]. Moreover, a mathematics department at a university may broadcast a collective opinion, such as through documentation on its website or through a hiring decision, that differs markedly from the opinions of its individual faculty members. The opinion of a research group, such as the applied-mathematics group, may also differ from the individual opinions of members of that group. In all of these examples, a group’s opinion does not necessarily reflect a consensus among the members of that group, as individuals in the group can disagree (sometimes rather strongly) with the group opinion. Importantly, groups and individual members

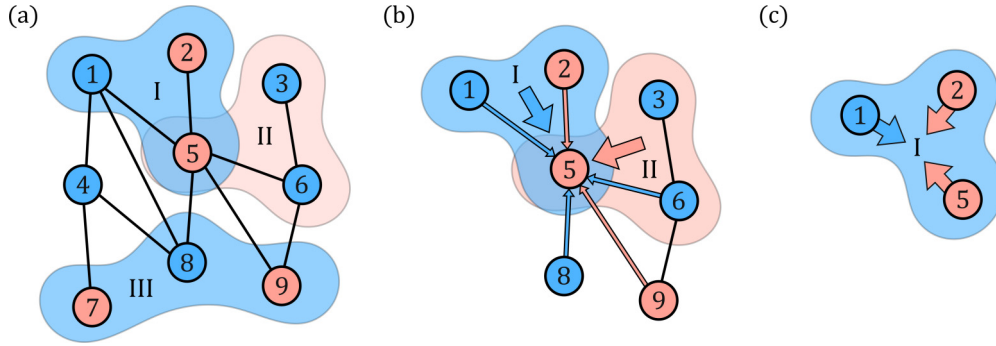


FIG. 1. A schematic illustration of how the opinions of nodes and groups are influenced by the opinions of other nodes and groups in our model. (a) A hypergraph with 9 nodes and 3 groups. Nodes 1, 3, 4, 6, and 8 have opinion 1 (in blue), and nodes 2, 5, 7, and 9 have opinion 0 (in red). Groups I and III have opinion 1 (in light blue), and group II has opinion 0 (in light red). (b) Node 5's opinion is influenced by the opinions of its neighboring nodes 1, 2, 6, 8, and 9 (thin arrows) and by the opinions of groups I and II (thick arrows). (c) Group I's opinion is influenced by the opinions of its constituent nodes 1, 2, and 5 (thick arrows).

of a group can influence each other. Group opinions are influenced by the opinions of their constituent members, and the opinions of individuals are influenced by the collective opinions of the groups in which they participate [30,31].

In the present paper, we formulate and analyze a model of opinion dynamics in which both a network's nodes and its groups of nodes have binary opinions. The opinions of individual nodes are affected both by their neighbors in a network and by the opinions of the groups in which they participate. The opinions of groups are affected by the opinions of their constituent nodes. For simplicity, we neglect interactions between distinct groups. In Fig. 1, we give a schematic illustration of how the opinions of nodes and groups influence the opinions of other nodes and groups in our model, which we describe in detail in Sec. II. In our model, group opinions can lead to oscillatory dynamics and excitable dynamics. In the oscillatory regime, the mean opinion of a network develops self-sustained oscillations. In the excitable regime, situations in which most of the nodes have the same opinion can quickly and temporarily change to situations in which most of the nodes have the other opinion. The emergence of oscillatory and excitable regimes depends strongly on the correlation between group structure and dyadic network adjacencies. We develop a mean-field approximation of our model's dynamics and use it to gain insight into it. We find that excitable dynamics arise because of a bifurcation from a stable equilibrium to sustained oscillatory dynamics. This change in qualitative dynamics resembles the onset of excitable dynamics in models of neuronal systems [32,33]. There is some qualitative similarity between the excitable dynamics in our model and the formation of social fads, which is a collective phenomenon in which a topic, object, or behavior experiences an increase in popularity that develops suddenly, lasts a short amount of time, and declines rapidly [34,35].

The role of group interactions, which are often called “higher-order interactions” or “polyadic interactions”, in opinion dynamics and other dynamical processes on networks has received much attention in the past few years [36–39]. The renewed interest in polyadic interactions in complex systems has built on foundational work that dates back many decades [40–43]. In social systems, a lot of recent research

has extended existing models to incorporate polyadic interactions [5,7,8,11,12,44–51]. In these extensions, the opinions are associated with individual agents and change due to both pairwise and group interactions. By contrast, in our model, we also assign opinions to the groups themselves. In this respect, our work is related to recent studies of the synchronization of quantities that are defined on the edges and higher-order simplices of a simplicial complex [52].

The incorporation of polyadic interactions into network dynamics can significantly influence the qualitative behavior of dynamical processes. For example, “opinion jumping” can occur in polyadic bounded-confidence models [8], bistable regions can arise in polyadic models of disease spread [44,53,54], and the qualitative nature of synchronization transitions of phase oscillators can be affected by polyadic interactions [55,56]. Our observation that group opinions can induce oscillatory and excitable dynamics provides another example of how polyadic interactions can fundamentally modify the dynamics of networked systems.

For simplicity, we consider polyadic interactions only through groups of size 3 (i.e., the groups always have exactly three nodes). Although this choice prevents us from exploring the effects of group-size heterogeneity (which occurs in most social systems), it has the important advantage of allowing us to easily engage with our primary focus, which is to study the effects of group opinions. Even with this major assumption, our model has very rich dynamics that differ markedly from existing opinion models (which do not incorporate group opinions).

In our model, the individual nodes do not account for their own current opinions when they update their opinions. With this choice of opinion updating, we emphasize the role of (both dyadic and polyadic) social interactions in opinion evolution. Essentially, we are examining a regime in which the effects of self-influence are small in comparison to the effects of social interactions. In this respect, our opinion model differs from many existing opinion models [16]; however, it follows the tradition of classical voter models [20–22]. In our model, groups can influence themselves, but distinct groups cannot influence each other. Because of group self-influence, we can use our model to explore the effects of

social phenomena, such as “pluralistic ignorance” [57,58] and “groupthink” [59], that can decelerate changes in group opinions. In pluralistic ignorance, the members of a group believe incorrectly that they hold a minority opinion within a group [57,58]. Pluralistic ignorance, which one can view as a “minority illusion” [60] in social dynamics, provides a potential mechanism to slow social change through a process of self-silencing, whereby individuals appear to conform to a belief that they believe is held by the rest of a group [61]. Groupthink refers to the tendency of individuals in a group to seek social conformity and thereby disregard their own opinions. Our opinion model does not directly encode pluralistic ignorance or groupthink, but one can view its group self-influence term (which slows changes in group opinions) as incorporating them indirectly.

Our paper proceeds as follows. In Sec. II, we introduce our stochastic model of opinion dynamics. In Sec. III, we discuss the model of random hypergraphs on which we simulate these opinion dynamics. In Sec. IV, we derive a mean-field description of our stochastic opinion model, study its steady-state behavior, and compare that behavior to the steady-state behavior of the original opinion model. We also discuss the assumptions and approximations that we use to derive the mean-field description. In Sec. V, we discuss the formation of group–node discordance states. In Sec. VI, we examine the formation of excitable and oscillatory dynamics. Finally, in Sec. VII, we summarize and discuss our findings. Our code, figures, and data are available online [62].

II. OUR STOCHASTIC OPINION MODEL

In this section, we describe our stochastic model of opinion dynamics with group opinions. We consider a set $\mathcal{V}$ of N nodes, which we index by $i \in \{1, 2, \dots, N\}$. Each node holds a binary opinion, which is either 0 or 1. Our model uses discrete time, so the time $t \in \{0, 1, \dots\}$. Let x_i^t denote the opinion of node i at time t . As in opinion models on ordinary graphs, which assume dyadic interactions between nodes, our nodes are adjacent to each other if there is an edge between them in a graph $\mathcal{G}$, which has an associated adjacency matrix A . We assume that $\mathcal{G}$ is undirected and unweighted, so $A_{ij} = A_{ji} = 1$ if nodes i and j are adjacent to each other and $A_{ij} = A_{ji} = 0$ if they are not adjacent. We also suppose that there are S groups of nodes; each group is a subset of $\mathcal{V}$. For simplicity, we suppose that all groups have exactly three nodes. Each group has either opinion 0 or opinion 1. We label the groups with the index $j \in \{1, 2, \dots, S\}$, and we denote the opinion of group j at time t by y_j^t . The $N \times S$ incidence matrix M has entries $M_{ij} = 1$ if node i participates in group j and $M_{ij} = 0$ if it does not. We often refer to the groups in our networks as “triangles”. The dyadic degree of each node is its number of edges, and the triadic degree of each node is its number of triangles. Node i ’s dyadic degree is thus $k_i = \sum_j A_{ij}$, and its triadic degree is $q_i = \sum_j M_{ij}$. The mean number of edges per node is $\langle k \rangle = \sum_i k_i / N$, and the mean number of triangles per node is $\langle q \rangle = \sum_i q_i / N$.

The opinion of a node is influenced both by the opinions of its adjacent nodes (i.e., dyadic influence) and by the opinions of the groups in which it participates (i.e., polyadic influence) [see Fig. 1(b)]. The opinion of a group is influenced by the

opinions of its constituent nodes [see Fig. 1(c)]. Groups do not directly influence other groups.

We now describe our model in detail. The opinions of the nodes (i.e., individuals) and triangles (i.e., groups) evolve stochastically according to the update rule

$$x_i^{t+1} = \begin{cases} 1 & \text{with probability } p_i^N(\mathbf{x}^t, \mathbf{y}^t) \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

$$y_j^{t+1} = \begin{cases} 1 & \text{with probability } p_j^E(\mathbf{x}^t, \mathbf{y}^t) \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where $\mathbf{x}^t = [x_1^t, x_2^t, \dots, x_N^t]^T$ is a node opinion vector at time t and $\mathbf{y}^t = [y_1^t, y_2^t, \dots, y_S^t]^T$ is a triangle opinion vector at time t .

Suppose that the probability that node i adopts opinion 1 is a nonlinear function of a linear combination of the opinions of its adjacent nodes and the opinions of the triangles in which it participates. That is,

$$p_i^N(\mathbf{x}^t, \mathbf{y}^t) = f_N(a\bar{x}_i^t + b\bar{y}_i^t), \quad (3)$$

where

$$\bar{x}_i^t = \sum_{j=1}^N A_{ij} x_j^t / \langle k \rangle, \quad (4)$$

$$\bar{y}_j^t = \sum_{k=1}^S M_{jk} y_k^t / \langle q \rangle, \quad (5)$$

the influence function f_N is a sigmoidal function [see Eq. (8) below], and a and b are real-valued constants.¹ The parameter a encodes the influence of nodes on their neighbors, and the parameter b encodes the influence of triangles on their constituent nodes. As we discussed in Sec. I, nodes do not consider their own current opinions when they update their opinions (i.e., $A_{ii} = 0$ for all i).

We suppose that the probability that a triangle adopts opinion 1 is a sigmoidal function of a linear combination of its own current opinion and the opinions of its constituent nodes. That is,

$$p_j^E(\mathbf{x}^t, \mathbf{y}^t) = f_E(c\bar{z}_j^t + d y_j^t), \quad (6)$$

where

$$\bar{z}_j^t = \frac{1}{3} \sum_{i=1}^N M_{ij} x_i^t, \quad (7)$$

the influence function f_E is a sigmoidal function [see Eq. (8) below], and c and d are real-valued constants. The parameter c encodes the influence of nodes on the triangles in which they participate, and the parameter d encodes the tendency of a triangle to maintain its opinion.

For simplicity, we use the same sigmoidal function for the influence function for all nodes and all triangles. We thus write

$$f(z) := f_N(z) = f_E(z) = \frac{1}{2} \{1 + \tanh[m(z - \mu)]\}, \quad (8)$$

¹One can absorb the quantities $\langle k \rangle$ and $\langle q \rangle$ in Eqs. (4) and (5) into the model parameters a , b , c , and d .

TABLE I. The parameters and other key quantities of our opinion model.

Parameter	Description
a	Influence of individuals on other individuals
b	Influence of groups on individuals
c	Influence of individuals on groups
d	Influence of a group on itself
f	Sigmoidal influence function
μ	Inflection point of the sigmoid f
$1/m$	Width of the transition of the sigmoid f
r	Correlation coefficient between dyadic degree and triadic degree
$\mathcal{P}$	Hyperdegree distribution
P_1	Marginal dyadic degree distribution
P_2	Marginal triadic degree distribution
k_i	Dyadic degree (i.e., ordinary node degree) of node i
q_i	Triadic degree of node i

where μ is the inflection point of the sigmoid $f(z)$ and $1/m$ is proportional to the width of the sigmoid's transition region. We use a sigmoidal function because it is convenient for representing saturating interactions [63]. Researchers have used sigmoidal functions in models of many other scenarios, including echo chambers and polarization [13], smooth bounded-confidence dynamics [14], and other saturating interactions, which occur in diverse fields that range from neuroscience to robotics [64].

In Table I, we summarize the parameters and other key quantities of our model. These parameters include the influence parameters (a , b , c , and d), the influence function f and its parameters μ and $1/m$, and parameters and other descriptors of network structure.

It is important to highlight the parameters a , b , c , and d , which encode the amount of (positive or negative) opinion influence at each time step. We interpret positive values of a , b , c , and d as conforming to influence, and we interpret negative values of these parameters as rejecting influence. The parameters a and b encode how much nodes are influenced by their neighboring nodes (the parameter a) and by the groups in which they participate (the parameter b). In particular, $a > 0$ (respectively, $a < 0$) increases (respectively, decreases) the probability that a node changes to or maintains opinion 1 when more of its neighbors have opinion 1. Analogously, $b > 0$ (respectively, $b < 0$) increases (respectively, decreases) the probability that a node changes to or maintains opinion 1 as it participates in more groups with opinion 1. The parameter c plays an analogous role for group opinions as b does for node opinions. Specifically, $c > 0$ (respectively, $c < 0$) increases (respectively, decreases) the probability that a group changes to or maintains opinion 1 when more of its participants have opinion 1. The parameter d encodes how much a group's current opinion affects its subsequent opinion. In particular, $d > 0$ (respectively, $d < 0$) increases (respectively, decreases) the probability that a group maintains opinion 1.

III. RANDOM-HYPERGRAPH MODEL

We use hypergraphs to describe our networks, which consist of nodes, edges, and triangles. A hypergraph is a generalization of a graph that includes both ordinary edges (i.e., dyadic adjacencies) and hyperedges with more than two nodes (i.e., polyadic adjacencies) [38,65]. Following standard convention, we refer to any of these adjacencies as “hyperedges”. Mathematically, a hypergraph $\mathcal{H}_{\mathcal{G}} = (\mathcal{V}, \mathcal{E})$ consists of a set $\mathcal{V}$ of nodes and a set $\mathcal{E}$ of hyperedges. Each hyperedge is a nonempty subset of $\mathcal{V}$; the number of nodes in this subset is the “size” of the hyperedge.

In an ordinary graph, each node $i \in \mathcal{V}$ has an associated degree k_i , which indicates the number of edges that are attached (i.e., “incident”) to it. In a hypergraph, the hyperdegree of node i is the vector $\mathbf{k}_i = [k_i^{(2)}, k_i^{(3)}, \dots, k_i^{(L)}]$, where L is the size of the hypergraph's largest hyperedge and the l th-order degree $k_i^{(l)}$ is the number of size- l hyperedges that are incident to the node. Each hypergraph has a hyperdegree distribution $\mathcal{P}(\mathbf{k})$, which encodes the probabilities that a uniformly-randomly-chosen node has hyperdegree $\mathbf{k}$ for each $\mathbf{k}$. We consider hypergraphs with hyperedges of sizes 2 and 3; the hyperedges of size 2 encode dyadic adjacencies, and the hyperedges of size 3 encode triadic (i.e., group) adjacencies. For such hypergraphs, the hyperdegree of a node is $\mathbf{k} = [k, q]$.

To study our opinion model, it is convenient to use random hypergraphs with specified hyperdegree sequences. We use such configuration-model random hypergraphs because we are able to control their hyperdegree sequences. In Appendix A, we provide a detailed discussion of the random-hypergraph model that we employ. The formation of hyperedges in this random-hypergraph model depends only on the specified hyperdegree of each node, so one can use hyperdegree-based compartmental models when studying dynamical processes on the hypergraphs that it generates. Such techniques have been used extensively in the study of disease spread on networks [66,67].

To examine the effects of correlations between the dyadic degree k and the triadic degree q , we use a convenient family of hyperdegree distributions to produce the degree sequences in our random hypergraphs. Given the marginal degree distributions $P_1(\cdot)$ and $P_2(\cdot)$ for the edges and triangles, respectively, there are two extremes of the joint distribution $\mathcal{P}(k, q)$. In one extreme, we let $k = q$, which implies that $\mathcal{P}(k, q) = P_1(k)\delta(k - q)$. In the other extreme, k and q are uncorrelated, which implies that $\mathcal{P}(k, q) = P_1(k)P_2(q)$. To systematically explore the effects of correlations between the dyadic and triadic degrees, we use a hyperdegree distribution that interpolates between these two extremes. This joint distribution is

$$\mathcal{P}(k, q) = P_1(k)P_2(q)(1 - r) + P_1(k)\delta(q - k)r, \quad (9)$$

where the Pearson correlation coefficient $r \in [0, 1]$ between the dyadic and triadic degrees parameterizes the amount of correlation between these degrees. When $r = 0$, the dyadic degree k and triadic degree q are uncorrelated; when $r = 1$, we have $k = q$ for every node.

To examine the effects of degree heterogeneity, we suppose that the marginal degree distributions $P_1(k)$ and $P_2(q)$ have the

approximate power-law form

$$P(k) := P_1(k) = P_2(k) = \begin{cases} \left(\frac{\gamma-1}{k_{\min}^{\gamma-1}}\right) k^{-\gamma}, & k \geq k_{\min} \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

where $k_{\min}$ is the minimum degree. In the hypergraphs that we construct using our random-hypergraph model, we generate the hyperdegree of each node using bivariate inverse sampling from the distribution that is described by Eqs. (9) and (10). We then construct our hypergraphs using the procedure that we describe in Appendix A.

IV. MEAN-FIELD APPROXIMATION, INITIAL CONDITIONS, AND STEADY-STATE SOLUTIONS

A. Mean-field approximation of Eqs. (1)–(2)

We develop a mean-field description that approximates the dynamics of our stochastic opinion model (1)–(2). Our mean-field description tracks the dynamics of three order parameters:

- (1) the expected fraction V^t of nodes with opinion 1 in a uniformly-randomly-selected edge at time t ;
- (2) the expected fraction U^t of nodes with opinion 1 in a uniformly-randomly-selected triangle at time t ; and
- (3) the expected fraction Y^t of triangles with opinion 1 at time t .

Alternatively, V^t represents the probability of moving to an opinion-1 node by following an edge that one chooses uniformly at random.

In this subsection, we present a simplified derivation of our mean-field approximation. In Appendix B, we show a detailed derivation of this approximation. Because we generate hypergraphs using a configuration model, the probability that there is a hyperedge that connects a group of nodes depends only on the hyperdegrees of those nodes. Therefore, we can use a hyperdegree-based compartmental model to obtain mean-field equations. First, we approximate the order parameters V^t and U^t in terms of the expected fraction $x_{\mathbf{k}}^t$ of nodes with hyperdegree $\mathbf{k} = [k, q]$ that have opinion 1 at time t . As we show in Appendix B, these approximations take the form

$$V^t = \sum_{\mathbf{k}} \frac{k \mathcal{P}(\mathbf{k}) x_{\mathbf{k}}^t}{\langle k \rangle}, \quad (11)$$

$$U^t = \sum_{\mathbf{k}} \frac{q \mathcal{P}(\mathbf{k}) x_{\mathbf{k}}^t}{\langle q \rangle}. \quad (12)$$

The variables V^t and U^t are both closely related to—but can differ from—the expected fraction $\sum_{\mathbf{k}} \mathcal{P}(\mathbf{k}) x_{\mathbf{k}}^t$ of nodes with opinion 1 at time t .

To obtain a system of discrete-time evolution equations for the order parameters V^t , U^t , and Y^t , consider the probability [see Eq. (1)] that a node i with hyperdegree $\mathbf{k} = [k, q]$ has opinion 1 at time $t + 1$. Assuming that all nodes with the same hyperdegree behave in the same way, we seek to approximate the variables $\bar{x}_i^t$, $\bar{y}_i^t$, $\bar{z}_j^t$, and y_j^t that appear in the probabilities (3)–(7) in terms of the order parameters.

We approximate $\bar{x}_i^t$, which is the normalized number of neighbors of node i that have opinion 1, by

$$\bar{x}_i^t \approx k V^t / \langle k \rangle \quad (13)$$

because node i is attached to approximately k edges and approximately a fraction V^t of these edges (see Appendix B) are attached to a node with opinion 1. The term $\bar{y}_i^t$, which is the normalized number of triangles that are attached to node i and have opinion 1 [see Eq. (7)], is approximately

$$\bar{y}_i^t \approx q Y^t / \langle q \rangle \quad (14)$$

because node i is attached to approximately q triangles and Y^t is the expected fraction of triangles that have opinion 1.

We insert the approximations (13) and (14) into Eqs. (1) and (3) to obtain

$$x_{\mathbf{k}}^{t+1} \approx f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right). \quad (15)$$

Under the mean-field assumption that all triangles behave in the same way (i.e., $y_j^t = Y^t$ and $\bar{z}_j^t = \bar{z}^t$ for all j), the time evolution of the expected fraction Y^t of triangles with opinion 1 satisfies

$$Y^{t+1} = f(c\bar{z}^t + d)Y^t + f(c\bar{z}^t)(1 - Y^t). \quad (16)$$

Similarly to our approximation of $\bar{y}_i^t$ in Eq. (14), we approximate $\bar{z}^t$ by

$$\bar{z}^t \approx U^t, \quad (17)$$

which is the expected fraction of nodes at time t with opinion 1 in a triangle that we select uniformly at random. Substituting Eq. (17) into Eq. (16) yields

$$Y^{t+1} = Y^t f(cU^t + d) + (1 - Y^t) f(cU^t). \quad (18)$$

Inserting Eq. (15) into Eqs. (11)–(12) yields a closed map for the time evolution of the three order parameters:

$$\begin{aligned} V^{t+1} &= \sum_{\mathbf{k}} \sum_q \frac{k \mathcal{P}(\mathbf{k}, q)}{\langle k \rangle} f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right), \\ U^{t+1} &= \sum_{\mathbf{k}} \sum_q \frac{q \mathcal{P}(\mathbf{k}, q)}{\langle q \rangle} f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right), \\ Y^{t+1} &= Y^t f(cU^t + d) + (1 - Y^t) f(cU^t). \end{aligned} \quad (19)$$

The mean-field description (19) relies on various approximations, which we now summarize and discuss. First, our mean-field description is a hyperdegree-based compartmental model, so it assumes that the expected time evolution of all nodes with hyperdegree $\mathbf{k}$ is the same. (For example, the probability that each such node has opinion 1 at time t is $x_{\mathbf{k}}^t$.) This approximation relies on the fact that we assume that all nodes with hyperdegree $\mathbf{k}$ possess the same type and number of expected connections. Our mean-field description also assumes that the dyadic and triadic degrees of each node are both sufficiently large that we can replace the variables $\bar{x}_i^t$, $\bar{y}_i^t$, and $\bar{z}_j^t$ by their means [as we did in Eqs. (13), (14), and (17)]. In partic-

ular, we do not expect our mean-field approximation to work well for sparse hypergraphs. One can generalize our mean-field description to account for hypergraph models (e.g., ones that generate degree-assortative random hypergraphs [68]) in which nodes have intrinsic variables and connect to each other with probabilities that depend on these variables. Such generalizations of configuration models have a long history of success in investigations of dynamical processes on graphs [69].

B. Selection of initial conditions

We now discuss our selection of initial conditions for our stochastic opinion model (1)–(2) and its mean-field approximation (19). The spaces of initial conditions in these two models differ drastically from each other, so we need to select initial conditions that allow us to compare these models as effectively as possible.

In our simulations of the stochastic opinion model (1)–(2), the initial opinion of each node and each triangle is either 0 or 1. To reduce the number of parameters that we need to describe the initial conditions of the stochastic opinion model, we specify only the initial probabilities that nodes and triangles have opinion 1. We specify the initial opinion of each node and each triangle independently. Each node initially has opinion 1 with probability u_1 and opinion 0 with probability $1 - u_1$, and each triangle initially has opinion 1 with probability u_2 and opinion 0 with probability $1 - u_2$. We then specify the initial conditions of the stochastic opinion model (1)–(2) as ordered pairs (u_1, u_2) of initial probabilities.

The mean-field map (19) describes the evolution of the three orders parameters V^t , U^t , and Y^t , so we directly specify the initial conditions V^0 , U^0 , and Y^0 of these order parameters. This specification contrasts with our selection of initial conditions in the stochastic opinion model (1)–(2), for which we can specify initial conditions using only two parameters. To mitigate this discrepancy, in our examples that compare simulations of (1)–(2) and (19), we let $V^0 = U^0 = u_1$ and $Y^0 = u_2$ to make the initial conditions for the two descriptions as similar as possible. We make this choice because the order parameters V^t and U^t are related (but not equal) to the fraction of nodes with opinion 1 [see Eqs. (11) and (12)] and Y^t is the expected fraction of triangles with opinion 1.

In several of our examples, we compare many possible choices of the initial conditions of the stochastic opinion model (1)–(2) and the mean-field map (19). In Appendix C, we give a detailed description of how we select these initial conditions.

C. Steady-state solutions of the stochastic opinion model (1)–(2)

We now examine the steady-state solutions (i.e., states in which the order parameters V^t , U^t , and Y^t are constant) of the stochastic opinion model (1)–(2) by studying the fixed points of the mean-field equations (19). For simplicity, we assume in the present discussion that the hyperdegree correlation is $r = 1$, which implies that the dyadic and triadic degrees are equal (i.e., $k = q$). We use this assumption, which restricts the possible dynamics, for the remainder of this section and throughout Sec. V. We will see in Sec. VI that relaxing this assumption leads to additional dynamics that are qualitatively

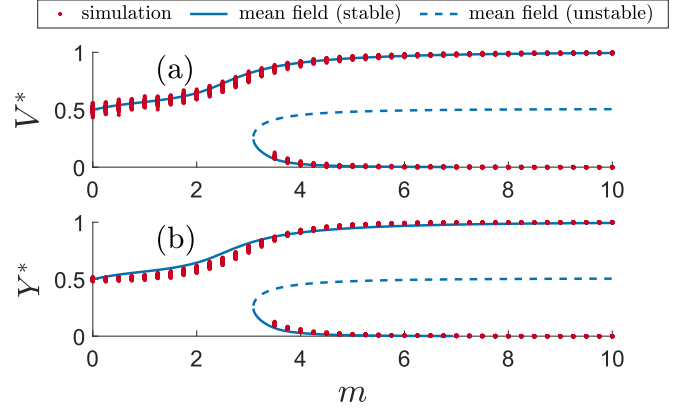


FIG. 2. An example of a bifurcation of the steady-state solutions when the dyadic and triadic degrees are equal (i.e., $r = 1$) in simulations of our stochastic opinion model (1)–(2) and solutions of the mean-field equations (20) for a single configuration-model hypergraph with $N = 700$ nodes, influence parameters $a = b = c = d = 0.5$, sigmoid inflection point $\mu = 0.5$, power-law exponent $\gamma = 4$, and mean degrees $\langle k \rangle = \langle q \rangle = 20$. The bifurcation parameter is the inverse-width parameter m of the sigmoidal influence function (8). We show the values of (a) V^* and (b) Y^* that we obtain from the mean-field equations (solid and dashed curves) and from means of 100 simulations of our stochastic opinion model (dots).

different from the dynamics that we study in this section and Sec. V. Under this assumption, $V^t = U^t$ and the mean-field equations (19) reduce to

$$\begin{aligned} V^{t+1} &= \sum_k \frac{kP(k)}{\langle k \rangle} f\left(\frac{k}{\langle k \rangle} [aV^t + bY^t]\right), \\ Y^{t+1} &= Y^t f(cV^t + d) + (1 - Y^t) f(cV^t). \end{aligned} \quad (20)$$

Any fixed point $[V^t, Y^t] = [V^*, Y^*]$ of the map (20) must satisfy

$$V^* = \sum_k \frac{kP(k)}{\langle k \rangle} f\left(\frac{k}{\langle k \rangle} [aV^* + bY^*]\right), \quad (21)$$

$$Y^* = Y^* f(cV^* + d) + (1 - Y^*) f(cV^*). \quad (22)$$

Solving (22) for Y^* and substituting the result into Eq. (21) shows that the fixed points of the map (20) have the form $[V^*, Y^*] = [F(V^*), G(V^*)]$, where

$$\begin{aligned} F(V) &= \sum_k \frac{kP(k)}{\langle k \rangle} f\left(\frac{k}{\langle k \rangle} [aV + bG(V)]\right), \\ G(V) &= \frac{f(cV)}{1 + f(cV) - f(cV + d)}. \end{aligned} \quad (23)$$

The equation $V^* = F(V^*)$ is a one-dimensional equation for V^* that one can solve using a root-finding algorithm. After determining V^* , one obtains Y^* using the equation $Y^* = G(V^*)$.

To illustrate the usefulness of Eqs. (21)–(22) to study steady-state solutions of the stochastic opinion model (1)–(2), we compare the fixed points that we obtain by solving Eqs. (21)–(22) to the results of simulations of Eqs. (1)–(2). In Fig. 2, we plot the steady-state values V^* and Y^* that we obtain from simulations of Eqs. (1)–(2) (dots) and the fixed-point solutions of Eqs. (21)–(22) (solid and dashed

curves) as functions of the sigmoid inverse-width parameter m for influence parameters $a = b = c = d = 0.5$, sigmoid inflection point $\mu = 0.5$, power-law exponent $\gamma = 4$, and mean degrees $\langle k \rangle = \langle q \rangle = 20$. Because a, b, c , and d all have positive values, all nodes and groups experience only conforming influence. For each value of m , we iterate Eqs. (1)–(2) for 400 steps and plot the values of V^* and Y^* after the final step. We do 100 independent simulations of this process with evenly spaced initial conditions in the unit square (see Appendix C). For each value of m , we use the same 700-node hypergraph.

Both our simulations of the stochastic opinion model (1)–(2) and our analysis of the mean-field approximation (20) illustrate that the system undergoes a transition from a regime with a single steady state to a regime with two distinct steady states as we increase m . Although we observe some quantitative differences between our mean-field description and direct simulations of the original stochastic opinion model, the fixed points of the mean-field equations are reasonably successful at approximating the steady-state solutions of the stochastic model. The bifurcation in Fig. 2 gives an interesting example of the behavior of our opinion model. A similar bifurcation was also observed in another opinion model with sigmoidal interactions [64]. Therefore, we do not focus on such bifurcations of steady-state solutions in situations with equal dyadic and triadic degrees ($r = 1$). Instead, we investigate novel features that arise due to the presence of group opinions. In particular, we observe (1) states in which the mean node opinions and mean group opinions are different and (2) excitable and oscillatory opinion dynamics. The oscillatory dynamics (see Sec. VI) arise only when the dyadic degree and triadic degree are not fully correlated (i.e., when $r < 1$).

V. GROUP-NODE DISCORDANCE

An important feature of our opinion model is that it admits solutions in which the mean opinion of the nodes differs significantly from the mean opinion of the groups. We refer to these solutions as *group-node discordance states*. These states can model situations in which a social organization (or other social group) has a different official stance than the individuals who comprise that organization. In our model, we measure the discordance of a solution by calculating

$$D(V^*, Y^*) = |V^* - Y^*|. \quad (24)$$

A group-node discordance state occurs when $D(V^*, Y^*) > 0$. The maximum possible discordance is $D(V^*, Y^*) = 1$.

We explore group-node discordance for different strengths of group influence (i.e., for different values of the group-influence parameters c and d). We plot $D(V^*, Y^*)$ versus the node-opinion influence parameter a (with $a = b$) using the mean values of V^* and Y^* from 16 independent simulations of the stochastic opinion model (1)–(2), and we compare this plot to a numerical solution of the fixed-point equations (21)–(22) for the mean-field approximation (20). In this comparison, we use a single realization of a configuration-model hypergraph with $N = 2000$ nodes, inverse-width parameter $m = 4$, power-law exponent $\gamma = 4$, and mean degrees $\langle k \rangle = \langle q \rangle = 20$ for both $\mu = 0.5$ [see Fig. 3(a)] and $\mu = 0.25$ [see Fig. 3(b)]. The initial conditions of the 16 independent simulations are evenly spaced in the unit square (see Appendix C). For a

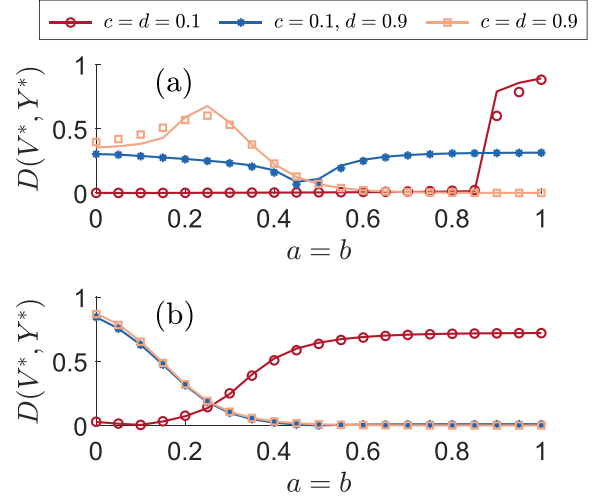


FIG. 3. The group-node discordance $D(V^*, Y^*)$ versus the node-opinion influence parameter a , with $a = b$, for a single numerical solution $(V^*, Y^*) = (F(V^*), G(Y^*))$ of Eqs. (23) (solid curves) and the mean of 16 independent simulations of the stochastic opinion model (1)–(2) (markers) for a single configuration-model hypergraph with $N = 2000$ nodes, inverse-width parameter $m = 4$, power-law exponent $\gamma = 4$, mean degrees $\langle k \rangle = \langle q \rangle = 20$, and several values of the group-influence parameters c and d . We consider sigmoid inflection points of (a) $\mu = 0.5$ and (b) $\mu = 0.25$. The initial conditions of the 16 simulations are evenly spaced in the unit square (see Appendix C).

sigmoid inflection point of $\mu = 0.5$, we obtain more group-node discordance [i.e., larger values of $D(V^*, Y^*)$] when the node parameters a and b are very different from the group parameters c and d . We see this in Fig. 3(a) for $c = d = 0.1$ (red curve and open circles) and $c = d = 0.9$ (orange curve and open squares). This is also the case for $\mu = 0.25$ [see Fig. 3(b)] when the group-influence parameters are $c = d = 0.1$ or $c = d = 0.9$. For both $\mu = 0.5$ and $\mu = 0.25$, the group-node discordance is most prominent when $c + d < \mu < a + b$ or $a + b < \mu < c + d$. For $\mu = 0.5$ with $c = 0.1$ and $d = 0.9$ [see the blue curve and blue stars in Fig. 3(a)], the amount of discordance does not vary much with a and b , except for a small decrease near $a = b = 0.5$. For $\mu = 0.5$, as we increase a , the transition to a group-node discordance state for $c = d = 0.1$ looks like it may be discontinuous. For $\mu = 0.25$, the transition between a group-node discordance state and a state without group-node discordance is continuous for all three examined parameter sets.

We also explore how the width (which is proportional to $1/m$) of the sigmoid transition region affects the onset of group-node discordance states by calculating $D(V^*, Y^*)$ versus m for several values of a, b, c , and d . We show the results of our numerical simulations in Fig. 4, which uses the same initial conditions, network parameters, and other conventions as Fig. 3. When $a = b = c = d = 0.5$ (blue curve and closed circles), the width parameter m has a minimal effect and $D(V^*, Y^*)$ remains close to 0, indicating that there is very little group-node discordance. For both $a = b = 0.2$, $c = d = 0.8$ (red curve and open circles) and $a = b = 0.8$, $c = d = 0.2$ (orange curve and open squares), the group-node discordance $D(V^*, Y^*)$ has a maximum at an intermediate

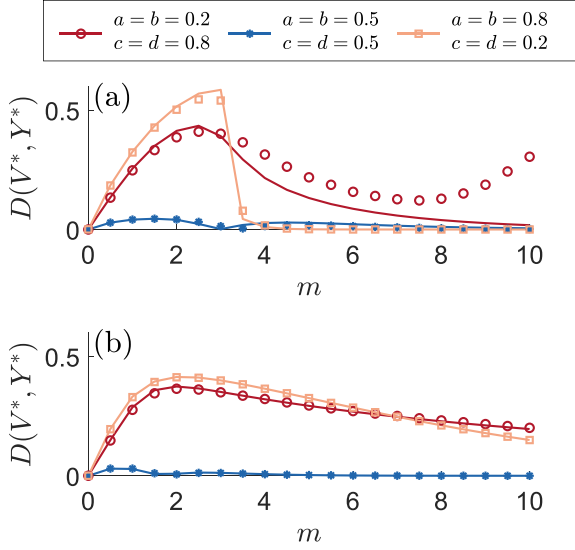


FIG. 4. The group-node discordance $D(V^*, Y^*)$ versus the sigmoid inverse-width parameter m for a single numerical solution $(V^*, Y^*) = (F(V^*), G(V^*))$ of Eqs. (23) (solid curves) and the mean of 16 independent simulations of the stochastic opinion model (1)–(2) (markers) for a single realization of a configuration-model hypergraph with $N = 2000$ nodes, power-law exponent $\gamma = 4$, mean degrees $\langle k \rangle = \langle q \rangle = 20$, and several values of a, b, c , and d . We consider (a) $\mu = 0.5$ and (b) $\mu = 0.25$. The initial conditions of the 16 simulations are evenly spaced in the unit square (see Appendix C).

value of m . We also observe an interesting difference between the cases $\mu = 0.5$ [see Fig. 4(a)] and $\mu = 0.25$ [see Fig. 4(b)]. When $a = b = 0.8$ and $c = d = 0.2$, there is a possibly discontinuous transition from discordance to non-discordance for $\mu = 0.5$; however, the transition is continuous (and gradual) for $\mu = 0.25$.

VI. EXCITABLE AND OSCILLATORY DYNAMICS

Our opinion model also has excitable and oscillatory opinion dynamics. To illustrate these dynamics, we simulate both the stochastic opinion model (1)–(2) and the mean-field approximation (19) with the parameters $(a, b, c, d) = (1, -0.5, 0.25, 0.25)$ and $(m, \mu) = (8, 0.25)$. In this regime, the nodes are influenced considerably by the opinions of their neighboring nodes ($a = 1$), nodes reject the opinions of their groups ($b = -0.5$), groups are influenced equally by their constituent nodes and their own opinions ($c = d = 0.25$), and the sigmoidal influence function of the nodes and groups has a small inflection point ($\mu = 0.25$) and a very steep transition ($m = 8$).

We first suppose that the dyadic and triadic degrees are equal (so the correlation between them is $r = 1$). This situation yields excitable dynamics, in which the system is initially at a locally stable steady-state solution, but—for a sufficiently large perturbation (which is often called a “stimulus”)—it experiences a large excursion through phase space before returning to the steady-state solution [32,33]. Excitable dynamics are common in neuronal and cardiac systems, and they are often associated with a system being near a bifurcation from a resting state to sustained spiking or oscillatory

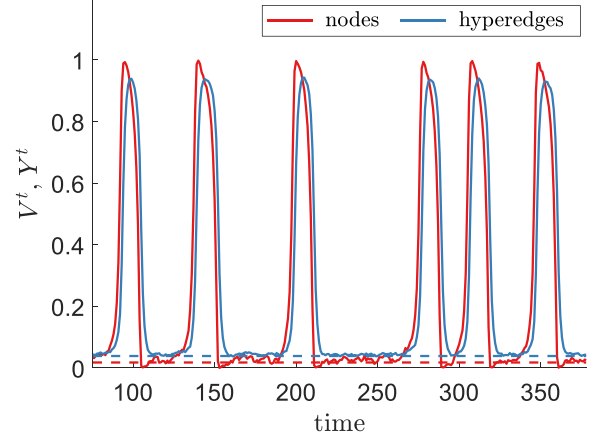


FIG. 5. An example of opinion pulses in a single simulation of our stochastic opinion model (1)–(2) with parameter values $a = 1$, $b = -0.5$, $c = d = 0.25$, $\mu = 0.25$, and $m = 8$ for a configuration-model hypergraph with equal dyadic and triadic degrees (i.e., $r = 1$) that we draw from an approximate power-law distribution with exponent $\gamma = 4$ and mean degrees $\langle k \rangle = \langle q \rangle = 20$. We plot the expected node fraction V^t in red and the expected triangle fraction Y^t in blue. The dashed lines show the fixed points that we obtain by solving Eqs. (21)–(22).

behavior [32,33,70]. In Fig. 5, we show the order parameters V^t (red) and Y^t (blue) from numerical simulations of Eqs. (1)–(2) for the aforementioned parameter values and a 1000-node configuration-model hypergraph with an approximate power-law hyperdegree distribution with $\gamma = 4$. The dashed curves show the fixed-point solution that we obtain by solving Eqs. (21)–(22). In these simulations, the expected node fraction V^t remains close to the fixed-point solution for a short time before it increases sharply and then subsequently decreases and returns approximately to the fixed-point solution. The expected triangle fraction Y^t has the same behavior; its dynamics follow V^t with a short delay. We use the term *opinion pulses* for these spikes in V^t and Y^t .

In Fig. 6, we compare a single pulse of the order parameter V^t in a simulation of the stochastic opinion model (1)–(2) (top) and the mean-field equations (19) (bottom). For the mean-field equations, we introduce a stimulus at time $t = 200$ by increasing both V^t and U^t by 0.2 (vertical arrow). The mean-field equations yield a single opinion pulse, which resembles the ones that we observe in simulations of our stochastic opinion model (1)–(2), in which finite-size fluctuations seemingly provide a stimulus.

As we decrease the correlation r between the dyadic and triadic degrees, the opinion pulses become more frequent until they eventually become self-sustained oscillations. In Fig. 7, we show an example of such oscillations for $r = 0.15$. We again observe that the mean-field equations (19) successfully reproduce the qualitative behavior of the stochastic opinion model (1)–(2).

One can use the mean-field equations (19) to understand the transition from excitable to oscillatory dynamics as one decreases the correlation r . When $r = 1$, the mean-field map (19) has three fixed points [see Fig. 8(a)]. We obtain these fixed points using root-finding methods and determine their linear stability by calculating the eigenvalues of the

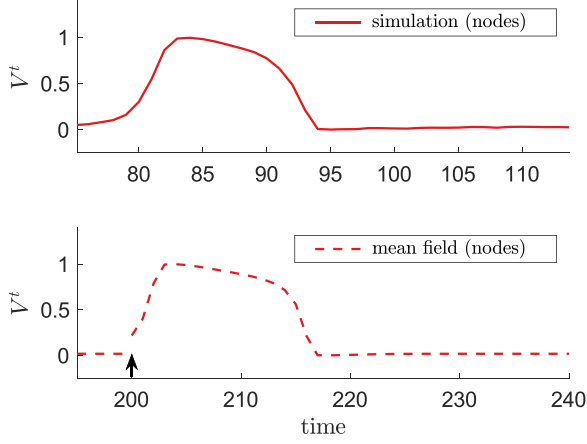


FIG. 6. Comparison of a single pulse of the order parameter V^t in (top) a single simulation of our stochastic opinion model (1)–(2) and (bottom) the mean-field equations (20) for the parameter values $a = 1$, $b = -0.5$, $c = d = 0.25$, $\mu = 0.25$, and $m = 8$ for a hypergraph with equal dyadic and triadic degrees (i.e., $r = 1$) that we draw from an approximate power-law distribution with exponent $\gamma = 4$ and mean degrees $\langle k \rangle = \langle q \rangle = 20$. We apply a stimulus (which is indicated by the black arrow) of $(\delta V, \delta U, \delta Y) = (0.2, 0.2, 0)$ to the mean-field equations to induce an excitation at time $t = 200$.

Jacobian matrix of Eq. (19). We compute the Jacobian matrix numerically using the “Adaptive Robust Numerical Differentiation” package for MATLAB [71]. One of the fixed points [see the closed blue circles in Figs. 8(a) and 8(b)] is linearly stable. It is located near the origin, so we refer to it as the “near-0” fixed point. A nearby fixed point [see the open green circles in Figs. 8(a)–8(c)] is a saddle. The third fixed point [see the red stars in Figs. 8(a)–8(c)] is an unstable

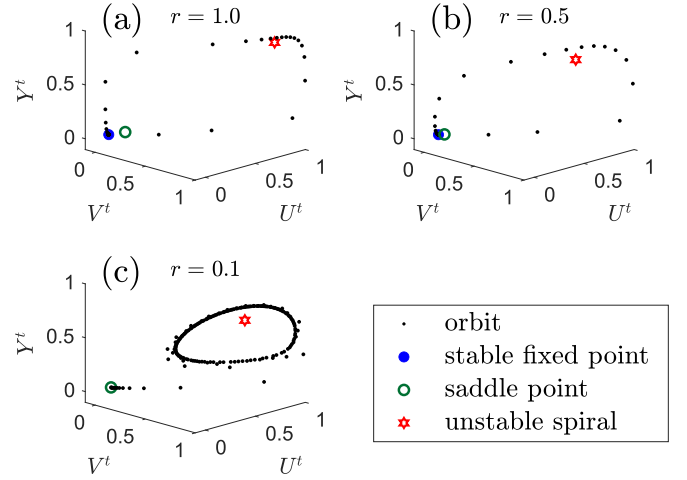


FIG. 8. The phase-space trajectories of solutions of the mean-field equations (19) with a perturbation $(\delta V, \delta U, \delta Y) = (0.25, 0.25, 0)$ for hyperdegree distributions with correlations between the dyadic and triadic degrees of (a) $r = 1$, (b) $r = 0.5$, and (c) $r = 0.1$. The merging of stable and unstable fixed points near an unstable spiral leads to the transition from excitable to oscillatory dynamics. The other parameter values are $a = 1$, $b = -0.5$, $c = d = 0.25$, $\mu = 0.25$, $m = 8$, $\gamma = 4$, and $\langle k \rangle = \langle q \rangle = 20$.

spiral, and we refer to it as the “away-from-0” fixed point. When the mean-field system (19) is close to the near-0 fixed point (which is linearly stable) and is perturbed so that it crosses the stable manifold of the saddle, it makes an excursion through phase space. It gets close to the unstable spiral and then returns to the stable fixed point, completing an opinion pulse. In Fig. 8(a), we show an example of such a trajectory in phase space. As we decrease r , the stable and saddle fixed points approach each other [see Fig. 8(b)], and one can then create opinion pulses using smaller stimuli. Eventually, the linearly stable fixed point and the saddle collide in a SNIC (which stands for “saddle–node on invariant cycle” and is pronounced “snick”) bifurcation, resulting in oscillatory behavior [see Fig. 8(c)]. See Ref. [33] for details about SNIC bifurcations.

We have seen that oscillatory dynamics arise via a SNIC bifurcation as we decrease the correlation coefficient r . To obtain a broader perspective on the bifurcations and the associated changes in qualitative dynamics that occur as we change the parameters, we show a bifurcation diagram in (r, γ) space in Fig. 9. Recall that γ is the exponent of the approximate power-law degree distributions of our configuration-model hypergraphs, so smaller values of γ correspond to more heterogeneous degree distributions. For small values of r and γ , the fixed points of the mean-field map (19) are linearly stable, which we indicate in the diagram by writing “steady state”. Through the numerical linear stability analysis that we described above, we observe three bifurcations as we increase γ and r (see Fig. 9). We describe these three bifurcations in the order that they occur. First, there is a Hopf bifurcation of the away-from-0 fixed point (red curve), which transitions from a linearly stable spiral to a linearly unstable spiral. This bifurcation marks the onset of oscillatory dynamics. Second, there is a SNIC bifurcation of the near-0

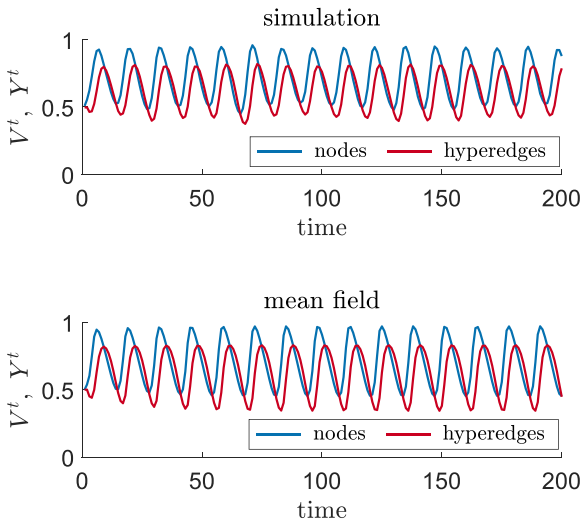


FIG. 7. An example of the oscillatory dynamics in (top) a single simulation of our stochastic opinion model (1)–(2) and (bottom) the mean-field equations (19) for parameter values $a = 1$, $b = -0.5$, $c = d = 0.25$, $\mu = 0.25$, $r = 0.15$, $m = 8$, $\gamma = 3.8$, and $\langle k \rangle = \langle q \rangle = 20$. We plot the expected node fraction V^t in blue and the expected triangle fraction Y^t in red. The initial conditions of the stochastic opinion model are $(u_1, u_2) = (0.5, 0.5)$, and the initial conditions of the mean-field equations are $(V^0, U^0, Y^0) = (0.5, 0.5, 0.5)$.

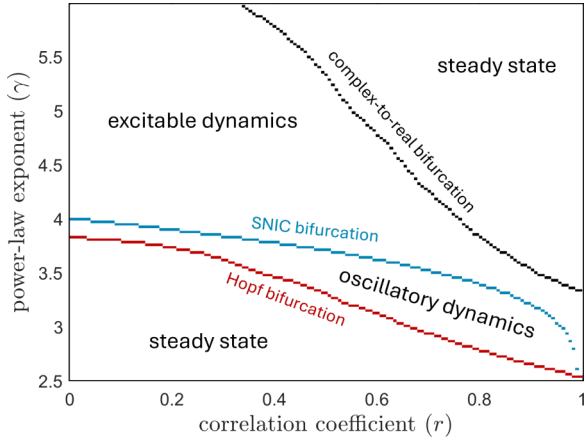


FIG. 9. A bifurcation diagram of the transitions between stationary, excitable, and oscillatory states of the mean-field equations (19). Each curve indicates a bifurcation in the linear stability of the near-0 fixed point or the away-from-0 fixed point. As one crosses the red curve, the away-from-0 fixed point undergoes a Hopf bifurcation. As one crosses the blue curve, the near-0 fixed point undergoes a saddle-node bifurcation. As one crosses the black curve, the away-from-0 fixed point transitions between a linearly unstable spiral and a linearly unstable node.

fixed point (blue curve), which transitions from a saddle to a linearly stable fixed point. This bifurcation marks a change from oscillatory dynamics to excitable dynamics. Third, there is a “complex-to-real bifurcation” where the away-from-0 fixed point transitions from an unstable spiral to an unstable node. This bifurcation marks the return of steady-state behavior. The bifurcation diagram in Fig. 9 indicates where excitable and oscillatory dynamics occur in the parameter range $(r, \gamma) \in [0, 1] \times [2.5, 6]$.

The mean-field map (19) is an approximation of the stochastic opinion model (1)–(2), so we expect similar bifurcations to occur in both sets of equations. Therefore, we believe that the aforementioned bifurcations provide a good explanation of the onset of excitable and oscillatory dynamics in both the mean-field equations (19) and the stochastic opinion model (1)–(2) that they approximate. To numerically verify this conjecture, we study the qualitative dynamics of Eqs. (1)–(2) for different values of both the correlation coefficient r and the exponent γ of the approximate power-law degree distribution. We compute the difference

$$\mathcal{H}(V^t, \mathcal{I}) := \max_{\mathcal{I}}(V^t) - \min_{\mathcal{I}}(V^t) \quad (25)$$

between the maximum and minimum values of the expected node fraction V^t in an interval $\mathcal{I}$. A fixed-point solution of the mean-field map (19) gives $\mathcal{H}(V^t, \mathcal{I}) \approx 0$ if we choose an interval $\mathcal{I}$ after the transient dynamics disappear. Oscillations and opinion pulses both yield $\mathcal{H}(V^t, \mathcal{I}) > 0$. In principle, one can also distinguish between pulses and oscillations by sliding and/or varying the length of the interval $\mathcal{I}$, but we do not employ these approaches (and we have not examined them thoroughly).

In Fig. 10, we plot $\mathcal{H}(V^t, \mathcal{I})$ versus the power-law exponent γ from Eq. (10) and the correlation coefficient r for dyadic and triadic degrees in the interval $\mathcal{I} = [100, 400]$,

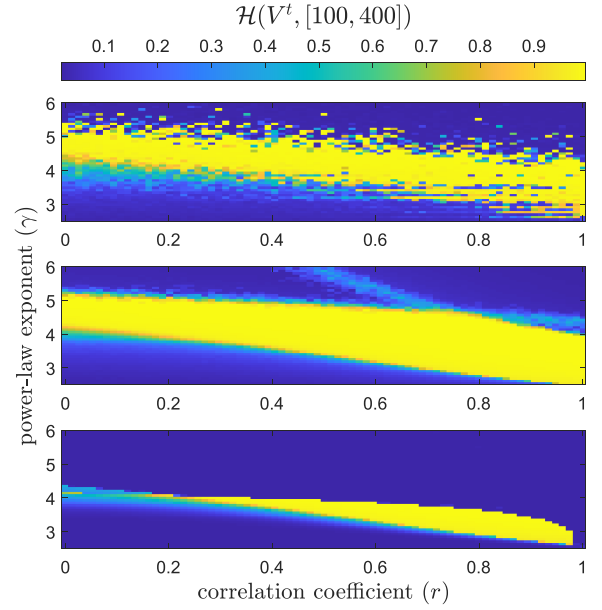


FIG. 10. A heat map of $\mathcal{H}(V^t, \mathcal{I}) = \max_{\mathcal{I}}(V^t) - \min_{\mathcal{I}}(V^t)$ [see Eq. (25)] for (top) a mean of 25 independent simulations of the stochastic opinion model (1)–(2), (middle) a mean of 25 independent simulations of the stochastically perturbed mean-field equations (26), and (bottom) a mean of 25 independent simulations of the original mean-field equations (19). The horizontal axis is the correlation coefficient r , and the vertical axis is the power-law exponent γ . In these simulations, we use a single realization of a configuration-model hypergraph and the parameter values $a = 1$, $b = -0.5$, $c = d = 0.25$, $\mu = 0.25$, $m = 8$, and $\langle k \rangle = \langle q \rangle = 20$. For both the stochastic opinion model and the mean-field equations (both with and without stochastic fluctuations), we select 25 evenly spaced initial conditions in the unit square (see Appendix C).

which seems to provide adequate time for the transient behavior to disappear. For each pair (r, γ) , we simulate the stochastic opinion model (1)–(2) and the mean-field equations (19) to obtain $\mathcal{H}$ from Eq. (25). For both the stochastic opinion model and the mean-field equations, we select 25 evenly spaced initial conditions in the unit square (see Appendix C). In the top panel of Fig. 10, we plot $\mathcal{H}$ from simulations of the stochastic opinion model (1)–(2). In the bottom panel, we plot $\mathcal{H}$ from the mean-field equations (19). For a given value of the correlation coefficient r , oscillatory or excitable dynamics occur only for a narrow range of power-law exponents, illustrating that these dynamics are very sensitive to network structure. A stronger correlation between dyadic and triadic degrees (i.e., a larger r) requires a smaller value of γ (i.e., a more heterogeneous network) for oscillatory or excitable dynamics to occur. Interestingly, perfectly correlated dyadic and triadic degrees (i.e., $r = 1$), which reduce the dimensionality of the mean-field equations (19) from 3 to 2, suppress the oscillatory dynamics. Observe that the yellow band [which indicates a large value of $\mathcal{H}(V^t, \mathcal{I})$] in the bottom panel of Fig. 10 does not extend to $r = 1$.

Despite the qualitative similarities between the dynamics of the stochastic opinion model (1)–(2) (see the top panel of Fig. 10) and those of the mean-field equations (19) (see the bottom panel of Fig. 10), there are key differences between

the qualitative dynamics of these models. As we discussed above, although the mean-field equations support excitable dynamics, they require a stimulus to yield opinion pulses. To mimic the effect of the stochastic model's finite-size fluctuations, which are absent in the deterministic mean-field equations (19), we introduce a stochastic term. We consider the equations

$$\begin{aligned} V^{t+1} &= \sum_k \sum_q \frac{k\mathcal{P}(k, q)}{\langle k \rangle} f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right) + \sigma_1^t, \\ U^{t+1} &= \sum_k \sum_q \frac{q\mathcal{P}(k, q)}{\langle q \rangle} f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right) + \sigma_2^t, \\ Y^{t+1} &= Y^t f(cU^t + d) + (1 - Y^t) f(cU^t). \end{aligned} \quad (26)$$

In the map (26), we draw σ_1^t and σ_2^t uniformly at random from the interval $(0, Q)$ at each time step. These small stochastic perturbations act as a repeated small stimulus to the system. Our decision to not include a random stimulus for Y^t arises from our observation that the fluctuations in the fraction of hyperedges with opinion 1 are smaller than the fluctuations in the fraction of nodes with opinion 1. This occurs because the hypergraphs that we generate using our random-hypergraph model have many more hyperedges than nodes. There are $N\langle q \rangle/3$ hyperedges in an N -node hypergraph.

In the middle panel of Fig. 10, we show $\mathcal{H}$ from the stochastically perturbed mean-field equations (26) with an upper bound of $Q \approx 0.036$ on the stochastic noise. We are not attempting to accurately reproduce the finite-size fluctuations of the stochastic opinion model (1)–(2). Instead, we seek to demonstrate that the mean-field description (19), when augmented with (admittedly ad hoc) stochastic fluctuations, can produce excitable and oscillatory dynamics that are qualitatively similar to those that we obtain from simulations of the stochastic opinion model.

By simultaneously examining Figs. 9 and 10, we observe that the mean-field approximation (see the bottom panel of Fig. 10) has a region with $\mathcal{H}(V^t, \mathcal{I}) > 0$ of a shape that is qualitatively similar to the oscillatory region in Fig. 9. We expect this similarity because our simulations of the mean-field equations (19) in the bottom panel of Fig. 10 do not include perturbations or stimuli, and they thus do not exhibit excitable dynamics. Additionally, in both the top and middle panels of Fig. 10, we observe that the region with $\mathcal{H}(V^t, \mathcal{I}) > 0$ is qualitatively similar to the union of the oscillatory region and the lower part of the excitable region in Fig. 9. When we use a larger value of Q in the stochastically perturbed mean-field map (26), the region with $\mathcal{H}(V^t, \mathcal{I}) > 0$ more closely resembles the union of the entirety of the excitable and oscillatory regions in Fig. 9.

VII. CONCLUSIONS AND DISCUSSION

We introduced and analyzed a stochastic model of opinion dynamics in which both nodes and groups of nodes have binary opinions. This opinion model includes novel dynamics that result directly from polyadic interactions. We showed that our model has a richer repertoire of qualitative dynamics than related models in which only nodes have opinions. In particular, our opinion model supports both excitable dynamics

(in which brief but strong opinion swings arise from perturbations of a steady-state solution) and oscillatory dynamics (in which the mean opinions of the nodes and groups have self-sustained oscillations). The excitable dynamics of our system has qualitative similarities to the dynamics of social fads [34]. In particular, a few individuals change their opinions initially, and then there is a surge in opinion changes in the system that quickly dies out. Our opinion model also possesses group–node discordance states, in which nodes and groups have contradictory opinions. Our simulations of our stochastic opinion model and its mean-field approximation both reveal that the excitable and oscillatory dynamics depend significantly on network structure (specifically, on dyadic degrees, polyadic degrees, and the correlation between them).

There are many interesting ways to extend our opinion model. As with all models of opinion dynamics, we greatly simplified human dynamics (or the dynamics of other animals) to formulate a mathematically and computationally tractable model that one can study systematically. For example, we assumed that opinions are binary (instead of allowing more opinion states or continuous-valued opinions), that interactions occur through a known and time-independent hypergraph, and that opinions evolve through precise mathematical rules. It is worth relaxing these assumptions and exploring the consequences of doing so.

One important way to generalize our model is to incorporate various heterogeneities, including in the group sizes, the interaction strengths (e.g., some groups or nodes may be more influential than others), and the shapes of the sigmoidal functions (e.g., some nodes may be more likely than others to change their opinions). For simplicity, we limited our study to groups of size 3. As we illustrated at length, the dynamics that result from considering only groups of size 3 is already very rich. However, it is natural to expect that some phenomena occur only in networks with heterogeneous group sizes. For example, perhaps an opinion can propagate from small groups to large groups (or vice versa). Just as the degree distribution of a graph can significantly influence the qualitative behavior of dynamical processes on it [65,72], we expect that the hyperedge-size distribution (along with hyperdegree distributions) influences the qualitative behavior of dynamical processes on a hypergraph. In our study, we also neglected interactions between distinct groups; such interactions are likely to introduce additional interesting dynamics. Another potentially interesting way to extend our model is to allow nodes to have self-influence, as individuals typically have some conviction in their prior beliefs. Additionally, although our mean-field approximation adequately reproduced the observed dynamics and provided some theoretical insights, it is based on the assumption that the hypergraph that describes the nodes and the groups is generated by a configuration model. It is worthwhile to extend our mean-field approximations to stochastic-block-model hypergraphs with assortative mixing, which can encode homophily [73] and community structure [74].

Although our opinion model has rich behavior and provides insights into the effects of group opinions on opinion dynamics, it is important to note that we have not validated our model with real-world opinion data. Such validation efforts are notoriously difficult in the study of opinion

dynamics [75,76], although they can be possible when appropriate data are available [77]. We hope that further studies of opinion dynamics will encourage and guide efforts in data collection, associated data analysis, and model validation.

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APPENDIX A: HYPERGRAPHS AND THE RANDOM-HYPERGRAPH MODEL

As we stated in Sec. III, it is convenient to use hypergraphs to describe our networks, which consist of nodes and groups of nodes. A hypergraph is a generalization of a graph that includes both ordinary edges (i.e., dyadic adjacencies) and hyperedges with more than two nodes (i.e., polyadic adjacencies) [38,64]. Following standard convention, we use the term “hyperedge” for any of these adjacencies. Mathematically, a hypergraph $\mathcal{H}_G = (\mathcal{V}, \mathcal{E})$ consists of a set $\mathcal{V}$ of nodes and a set $\mathcal{E}$ of hyperedges. Each hyperedge is a nonempty subset of $\mathcal{V}$; the number of nodes in this subset is the “size” of the hyperedge. In an ordinary graph, each node $i \in \mathcal{V}$ has an associated degree k_i , which indicates the number of edges that are attached (i.e., “incident”) to it. The “hyperdegree” of node i is the vector $\mathbf{k}_i = [k_i^{(2)}, k_i^{(3)}, \dots, k_i^{(L)}]$, where L is the size of its largest hyperedge and the l th-order degree $k_i^{(l)}$ is the number of size- l hyperedges that are incident to node i . Each hypergraph has a hyperdegree distribution $\mathcal{P}(\mathbf{k})$, which encodes the probabilities that a uniformly-randomly-chosen node has hyperdegree $\mathbf{k}$ for each $\mathbf{k}$.

Just as one can describe a hypergraph using a bipartite network [65], it is also possible to formulate our opinion model by considering dynamics on an ordinary graph with two types of nodes. In this formulation, the agents constitute one type of node and the groups constitute a second type of node. An agent can have adjacencies both with groups and with other agents, and a group is adjacent to each agent that participates in it. We view the group language as much more natural than bipartite-network language, just as the language of hypergraphs and simplicial complexes is natural for studying polyadic interactions [38,78].

We now describe the particular random-hypergraph model that we employ. Consider a set of nodes $i \in \{1, 2, 3, \dots, N\}$ with hyperdegrees $\mathbf{k}_i$, where $k_i^{(l)}$ is the number of loose ends (i.e., “stubs”) of size- l hyperedges that are attached to i . We form a size- l hyperedge by uniformly randomly selecting l stubs for the hyperedge. We repeat this stub-selection process until all stubs are assigned to a hyperedge. If the mean $\langle k^{(l)} \rangle$ is finite, the probability that there is a size- l hyperedge that

connects nodes $\{i_1, i_2, \dots, i_l\}$ in the limit $N \rightarrow \infty$ is

$$f_l(\mathbf{k}_{i_1}, \mathbf{k}_{i_2}, \dots, \mathbf{k}_{i_l}) = \frac{(l-1)! k_{i_1}^{(l)} k_{i_2}^{(l)} \times \dots \times k_{i_l}^{(l)}}{(N \langle k^{(l)} \rangle)^{l-1}}. \quad (\text{A1})$$

This expression is a generalization of an associated expression for ordinary configuration-model graphs [65,79].

The above random-hypergraph model is a special case of the stub-labeled hypergraph configuration model in [68] in which we allow only hyperedges of sizes 2 and 3. Constructing a hypergraph with a configuration model yields a small number of self-hyperedges (i.e., hyperedges in which a single node participates two or more times) and multi-hyperedges (i.e., redundant hyperedges, which are hyperedges that occur two or more times). See Sec. 12.1 of Ref. [65] and Ref. [79] for relevant discussions in the context of graphs. Because we assume that nodes do not influence themselves, we remove self-hyperedges when constructing hypergraphs. For example, in one sample of 20 instantiations of a 2500-node hypergraph with $\gamma = 4$ and $\langle k \rangle = \langle q \rangle = 20$, we removed approximately 0.05% of the edges and approximately 0.15% of the triangles. In our computations, we retain multi-hyperedges. In the same sample of 20 instantiations of a 2500-node hypergraph with $\gamma = 4$ and $\langle k \rangle = \langle q \rangle = 20$, about 0.33% of all edges and about 0.0003% of all triangles are multi-hyperedges.

Our random-hypergraph model is reminiscent of the closely related random-graph models with clustering that were proposed independently by Newman [80] and Miller [81]. In a Newman–Miller model, nodes have specified dyadic and triadic (i.e., triangle) degree sequences, which one uses independently in a stub-matching procedure. However, a “triangle” in the Newman–Miller model corresponds to three dyadic interactions, rather than to a single size-3 hyperedge.

APPENDIX B: DETAILED DERIVATION OF THE MEAN-FIELD APPROXIMATION (19)

In this appendix, we give a detailed derivation of our mean-field equations (19). As we discussed in Sec. IV A, we consider the time evolution of the three order parameters V^t , U^t , and Y^t .

We begin by expressing V^t and U^t in terms of $x_{\mathbf{k}}^t$, which is the fraction of nodes at time t with hyperdegree $\mathbf{k} = [k, q]$ that have opinion 1. The total number of edges is $N \langle k \rangle / 2$, and the total number of triangles is $N \langle q \rangle / 3$. The expected fraction of opinion-1 nodes in a uniformly-randomly-selected edge is thus

$$\begin{aligned} V^t &= \sum_{\mathbf{k}} \sum_{\mathbf{k}'} \frac{NP(\mathbf{k})NP(\mathbf{k}')}{2!} \left(\frac{kk'}{N \langle k \rangle} \right) \left(\frac{x_{\mathbf{k}}^t + x_{\mathbf{k}'}^t}{2} \right) / \left(\frac{N \langle k \rangle}{2} \right) \\ &= \sum_{\mathbf{k}} \frac{k \mathcal{P}(\mathbf{k}) x_{\mathbf{k}}^t}{\langle k \rangle}, \end{aligned} \quad (\text{B1})$$

where $NP(\mathbf{k})NP(\mathbf{k}')/2!$ is the expected number of pairs of nodes with hyperdegrees $\mathbf{k}$ and $\mathbf{k}'$, the quantity $kk'/(N \langle k \rangle)$ is the expected fraction of these pairs that are connected by an edge [see Eq. (A1) with $l = 2$], and $(x_{\mathbf{k}}^t + x_{\mathbf{k}'}^t)/2$ is the expected fraction of opinion-1 nodes that are attached to an edge that connects uniformly-randomly-selected nodes with

hyperdegrees $\mathbf{k}$ and $\mathbf{k}'$. Analogously, the expected fraction of opinion-1 nodes in a uniformly-randomly-selected triangle is

$$\begin{aligned} U^t &= \sum_{\mathbf{k}} \sum_{\mathbf{k}'} \sum_{\mathbf{k}''} \frac{N\mathcal{P}(\mathbf{k})N\mathcal{P}(\mathbf{k}')N\mathcal{P}(\mathbf{k}'')}{3!} \left(\frac{2q q' q''}{(N\langle q \rangle)^2} \right) \left(\frac{x_{\mathbf{k}}^t + x_{\mathbf{k}'}^t + x_{\mathbf{k}''}^t}{3} \right) / \left(\frac{N\langle q \rangle}{3} \right) \\ &= \sum_{\mathbf{k}} \frac{q\mathcal{P}(\mathbf{k})x_{\mathbf{k}}^t}{\langle q \rangle}. \end{aligned} \quad (\text{B2})$$

Because we generate hypergraphs using a configuration model, the probability that there is a hyperedge that connects a group of nodes depends only on the hyperdegrees of those nodes. Therefore, we are able to use a hyperdegree-based compartmental model to derive mean-field equations. Consider the probability [see Eq. (1)] that a node i with hyperdegree $\mathbf{k} = [k, q]$ has opinion 1 at time $t + 1$. Assuming that all nodes with the same hyperdegree behave in the same way (i.e., the probability that node j has opinion 1 is $x_{\mathbf{k}}^t$ for all nodes j with hyperdegree $\mathbf{k}_j = \mathbf{k}$), we approximate the normalized number $\bar{x}_i^t$ of neighbors of node i that have opinion 1 [see Eq. (4)] by

$$\begin{aligned} \bar{x}_i^t &= \frac{1}{\langle k \rangle} \sum_{j=1}^N A_{ij} x_j^t \\ &\approx \frac{1}{\langle k \rangle} \sum_{\mathbf{k}'} N\mathcal{P}(\mathbf{k}', q') \left(\frac{k k'}{N\langle k \rangle} \right) x_{\mathbf{k}'}^t \\ &= \frac{k}{\langle k \rangle} \sum_{\mathbf{k}'} \frac{k' \mathcal{P}(\mathbf{k}', q') x_{\mathbf{k}'}^t}{\langle k \rangle} = k V^t / \langle k \rangle, \end{aligned} \quad (\text{B3})$$

where the approximation in the second line replaces the number of opinion-1 neighbors of node i with its expected value. The term $\bar{y}_i^t$, which is the normalized number of triangles that are attached to node i and have opinion 1 [see Eq. (7)], is approximately

$$\bar{y}_i^t \approx q Y^t / \langle q \rangle \quad (\text{B4})$$

because node i is attached to q triangles and Y^t is the expected fraction of triangles that have opinion 1. We insert the approximations (B3) and (B4) into Eqs. (1) and (3) to obtain

$$\begin{aligned} x_{\mathbf{k}}^{t+1} &= \frac{1}{N\mathcal{P}(\mathbf{k})} \sum_{\mathbf{k}_i=\mathbf{k}} \mathbb{E}[x_i^{t+1}] = \frac{1}{N\mathcal{P}(\mathbf{k})} \sum_{\mathbf{k}_i=\mathbf{k}} f(a\bar{x}_i^t + b\bar{y}_i^t) \\ &\approx \frac{1}{N\mathcal{P}(\mathbf{k})} \sum_{\mathbf{k}_i=\mathbf{k}} f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right) \\ &= f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right), \end{aligned} \quad (\text{B5})$$

where $\mathbb{E}[\cdot]$ denotes the expectation. The time evolution of the expected fraction Y^t of triangles with opinion 1 satisfies

$$\begin{aligned} Y^{t+1} &= \mathbb{E}\left[\frac{1}{S} \sum_{j=1}^S y_j^{t+1}\right] = \frac{1}{S} \sum_{j=1}^S \mathbb{E}[y_j^{t+1}] \\ &= \frac{1}{S} \sum_{j=1}^S \{\mathbb{E}[y_j^{t+1} | y_j^t = 1] P(y_j^t = 1) + \mathbb{E}[y_j^{t+1} | y_j^t = 0] P(y_j^t = 0)\}. \end{aligned} \quad (\text{B6})$$

Making the mean-field assumption that all triangles behave in the same way (i.e., $y_j^t = y^t$ and $\bar{z}^t = \bar{z}_j^t$ for all j) yields

$$\begin{aligned} Y^{t+1} &= \mathbb{E}[y^{t+1} | y^t = 1] P(y^t = 1) \\ &\quad + \mathbb{E}[y^{t+1} | y^t = 0] P(y^t = 0). \end{aligned} \quad (\text{B7})$$

We then use Eq. (6) for the expected values and the relation $P(y^t = 1) = Y^t$ to obtain

$$Y^{t+1} = f(c\bar{z}^t + d)Y^t + f(c\bar{z}^t)(1 - Y^t). \quad (\text{B8})$$

Finally, similarly to our approximation of $\bar{y}_i^t$ in Eq. (14), we approximate $\bar{z}^t$ (i.e., the fraction of nodes with opinion 1 in a triangle that we select uniformly at random) by setting

$$\bar{z}^t \approx U^t, \quad (\text{B9})$$

which is the expected fraction of opinion-1 nodes at time t in a triangle that we select uniformly at random. Substituting Eq. (B9) into Eq. (B8) yields

$$Y^{t+1} = Y^t f(cU^t + d) + (1 - Y^t) f(cU^t). \quad (\text{B10})$$

Inserting Eq. (B5) into Eqs. (B1)–(B2) yields the closed map (19) for the time evolution of the three order parameters:

$$\begin{aligned} V^{t+1} &= \sum_{\mathbf{k}} \sum_{\mathbf{q}} \frac{k\mathcal{P}(\mathbf{k}, q)}{\langle k \rangle} f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right), \\ U^{t+1} &= \sum_{\mathbf{k}} \sum_{\mathbf{q}} \frac{q\mathcal{P}(\mathbf{k}, q)}{\langle q \rangle} f\left(\frac{ak}{\langle k \rangle} V^t + \frac{bq}{\langle q \rangle} Y^t\right), \\ Y^{t+1} &= Y^t f(cU^t + d) + (1 - Y^t) f(cU^t). \end{aligned} \quad (\text{B11})$$

APPENDIX C: SELECTION OF INITIAL CONDITIONS

In this appendix, we discuss how we select initial conditions for our simulations of the stochastic opinion model (1)–(2), the deterministic mean-field approximation (19), and the stochastically perturbed mean-field approximation (26). In most of our simulations, we perform parameter sweeps using W^2 evenly spaced initial conditions in the unit square. For each initial condition, we perform a single simulation of the

stochastic opinion model (1)–(2), the deterministic mean-field map (19), or the stochastically perturbed mean-field map (26).

For the stochastic opinion model (1)–(2), we use the W^2 evenly spaced probabilities (u_1, u_2) (which we described in Sec. IV B) in the set $\{(i/(W-1), j/(W-1)) | i, j \in \{0, 1, \dots, W-1\}\}$ for our parameter sweeps. For the mean-field equations (19) and (26), we need to select initial values

(V^0, U^0, Y^0) of the three order parameters. As we described in Sec. IV B, we employ initial conditions with $V^0 = U^0$ so that our initial conditions resemble our initial conditions for the stochastic opinion model as closely as possible. Therefore, we use the W^2 evenly spaced initial conditions (V^0, U^0, Y^0) in the set $\{(i/(W-1), i/(W-1), j/(W-1)) | i, j \in \{0, 1, \dots, W-1\}\}$ for our parameter sweeps.

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- [1] I. V. Kozitsin, Opinion dynamics of online social network users: A micro-level analysis, *J. Math. Soc.* **47**, 1 (2023).
 - [2] Q. Zha, G. Kou, H. Zhang, H. Liang, X. Chen, C.-C. Li, and Y. Dong, Opinion dynamics in finance and business: A literature review and research opportunities, *Financ. Innov.* **6**, 44 (2020).
 - [3] J. Fernández-Gracia, K. Suchecki, J. J. Ramasco, M. San Miguel, and V. M. Eguíluz, Is the voter model a model for voters? *Phys. Rev. Lett.* **112**, 158701 (2014).
 - [4] D. A. Siegel, Social networks and collective action, *Am. J. Polit. Sci.* **53**, 122 (2009).
 - [5] L. Horstmeyer and C. Kuehn, Adaptive voter model on simplicial complexes, *Phys. Rev. E* **101**, 022305 (2020).
 - [6] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch, Mixing beliefs among interacting agents, *Adv. Complex Syst.* **03**, 87 (2000).
 - [7] L. Neuhäuser, A. Mellor, and R. Lambiotte, Multibody interactions and nonlinear consensus dynamics on networked systems, *Phys. Rev. E* **101**, 032310 (2020).
 - [8] A. Hickok, Y. Kureh, H. Z. Brooks, M. Feng, and M. A. Porter, A bounded-confidence model of opinion dynamics on hypergraphs, *SIAM J. Appl. Dyn. Syst.* **21**, 1 (2022).
 - [9] A. Srivastava, C. Chelmiss, and V. K. Prasanna, Computing competing cascades on signed networks, *Soc. Netw. Anal. Min.* **6**, 82 (2016).
 - [10] C. Toccaceli, L. Milli, and G. Rossetti, Opinion dynamic modeling of fake news perception, in *Complex Networks & Their Applications IX. COMPLEX NETWORKS 2020*, edited by R. M. Benito, C. Cherifi, H. Cherifi, E. Moro, L. Rocha, and M. Sales-Pardo (Springer-Verlag, Heidelberg, Germany, 2020), pp. 370–381.
 - [11] S. Maletić and M. Rajković, Consensus formation on a simplicial complex of opinions, *Physica A* **397**, 111 (2014).
 - [12] R. Sahasrabudde, L. Neuhäuser, and R. Lambiotte, Modelling non-linear consensus dynamics on hypergraphs, *J. Phys. Complex.* **2**, 025006 (2021).
 - [13] F. Baumann, P. Lorenz-Spreen, I. M. Sokolov, and M. Starnini, Modeling echo chambers and polarization dynamics in social networks, *Phys. Rev. Lett.* **124**, 048301 (2020).
 - [14] H. Z. Brooks, P. S. Chodrow, and M. A. Porter, Emergence of polarization in a sigmoidal bounded-confidence model of opinion dynamics, *SIAM J. Appl. Dyn. Syst.* **23**, 1442 (2024).
 - [15] A. Sîrbu, V. Loreto, V. D. P. Servedio, and F. Tria, Opinion dynamics: Models, extensions and external effects, in *Participatory Sensing, Opinions and Collective Awareness*, edited by V. Loreto, M. Haklay, A. Hotho, V. D. Servedio, G. Stumme, J. Theunis, and F. Tria (Springer International Publishing, Cham, Switzerland, 2017), pp. 363–401.
 - [16] H. Noorazar, K. R. Vixie, A. Talebanpour, and Y. Hu, From classical to modern opinion dynamics, *Int. J. Mod. Phys. C* **31**, 2050101 (2020).
 - [17] H. Olsson and M. Galesic, Analogies for modeling belief dynamics, *Trends Cognit. Sci.* **28**, 907 (2024).
 - [18] Z. Xie, X. Song, and Q. Li, A review of opinion dynamics, in *Theory, Methodology, Tools and Applications for Modeling and Simulation of Complex Systems: 16th Asia Simulation Conference and SCS Autumn Simulation Multi-Conference, AsiaSim/SCS AutumnSim 2016*, edited by L. Zhang, X. Song, and Y. Wu, Proceedings, Part IV (Springer-Verlag, Heidelberg, Germany, 2016), pp. 349–357.
 - [19] M. H. DeGroot, Reaching a consensus, *J. Am. Stat. Assoc.* **69**, 118 (1974).
 - [20] P. Clifford and A. Sudbury, A model for spatial conflict, *Biometrika* **60**, 581 (1973).
 - [21] R. A. Holley and T. M. Liggett, Ergodic theorems for weakly interacting infinite systems and the voter model, *Ann. Probab.* **3**, 643 (1975).
 - [22] S. Redner, Reality-inspired voter models: A mini-review, *C. R. Phys.* **20**, 275 (2019).
 - [23] S. Galam, Minority opinion spreading in random geometry, *Eur. Phys. J. B* **25**, 403 (2002).
 - [24] C. Bernardo, C. Altafini, A. Proskurnikov, and F. Vasca, Bounded confidence opinion dynamics: A survey, *Automatica* **159**, 111302 (2024).
 - [25] R. L. Breiger, The duality of persons and groups, *Soc. Forces* **53**, 181 (1974).
 - [26] C. Burningham and M. A. West, Individual, climate, and group interaction processes as predictors of work team innovation, *Small Group Res.* **26**, 106 (1995).
 - [27] G. A. Fine, The hinge: Civil society, group culture, and the interaction order, *Soc. Psych. Q.* **77**, 5 (2014).
 - [28] M. Eilert and A. Nappier Cherup, The activist company: Examining a company's pursuit of societal change through corporate activism using an institutional theoretical lens, *J. Public Policy Mark.* **39**, 461 (2020).
 - [29] R. C. Black, R. J. Owens, J. Wedeking, and P. C. Wohlfarth, *US Supreme Court Opinions and Their Audiences* (Cambridge University Press, Cambridge, UK, 2016).
 - [30] R. B. Cialdini and N. J. Goldstein, Social influence: Compliance and conformity, *Annu. Rev. Psychol.* **55**, 591 (2004).
 - [31] V. Griskevicius, N. J. Goldstein, C. R. Mortensen, R. B. Cialdini, and D. T. Kenrick, Going along versus going alone: When fundamental motives facilitate strategic (non)conformity, *J. Pers. Soc. Psych.* **91**, 281 (2006).
 - [32] R. Sepulchre, G. Drion, and A. Franci, Excitable behaviors, in *Emerging Applications of Control and Systems Theory: A Festschrift in Honor of Mathukumalli Vidyasagar* (Springer-Verlag, Heidelberg, Germany, 2018), pp. 269–280.
 - [33] E. M. Izhikevich, *Dynamical Systems in Neuroscience* (MIT Press, Cambridge, MA, USA, 2007).

- [34] J. Best, *Flavor of the Month: Why Smart People Fall for Fads* (University of California Press, Oakland, CA, USA, 2006).
- [35] B. E. Aguirre, E. L. Quarantelli, and J. L. Mendoza, The collective behavior of fads: The characteristics, effects, and career of streaking, *Am. Soc. Rev.* **53**, 569 (1988).
- [36] F. Battiston, G. Cencetti, I. Iacopini, V. Latora, M. Lucas, A. Patania, J.-G. Young, and G. Petri, Networks beyond pairwise interactions: Structure and dynamics, *Phys. Rep.* **874**, 1 (2020).
- [37] F. Battiston, E. Amico, A. Barrat, G. Bianconi, G. Ferraz de Arruda, B. Franceschiello, I. Iacopini, S. Kéfi, V. Latora, Y. Moreno, M. M. Murray, T. P. Peixoto, F. Vaccarino, and G. Petri, The physics of higher-order interactions in complex systems, *Nat. Phys.* **17**, 1093 (2021).
- [38] C. Bick, E. Gross, H. A. Harrington, and M. T. Schaub, What are higher-order networks? *SIAM Rev.* **65**, 686 (2023).
- [39] Z. Gao, D. Ghosh, H. A. Harrington, J. G. Restrepo, and D. Taylor, Dynamics on networks with higher-order interactions, *Chaos* **33**, 040401 (2023).
- [40] R. H. Atkin, *Combinatorial Connectivities in Social Systems: An Application of Simplicial Complex Structures to the Study of Large Organizations* (Springer-Verlag, Heidelberg, Germany, 1977).
- [41] P. A. Abrams, Arguments in favor of higher order interactions, *Am. Nat.* **121**, 887 (1983).
- [42] H. M. Wilbur and J. E. Fauth, Experimental aquatic food webs: Interactions between two predators and two prey, *Am. Nat.* **135**, 176 (1990).
- [43] H. Schaffer and C. Liddell, Adult-child interaction under dyadic and polyadic conditions, *Br. J. Dev. Psych.* **2**, 33 (1984).
- [44] I. Iacopini, G. Petri, A. Barrat, and V. Latora, Simplicial models of social contagion, *Nat. Commun.* **10**, 2485 (2019).
- [45] L. Neuhäuser, M. T. Schaub, A. Mellor, and R. Lambiotte, Opinion dynamics with multi-body interactions, in *International Conference on Network Games, Control and Optimization: NETGCOOP 2021*, edited by S. Lasaulce, P. Mertikopoulos, and A. Orda (Springer International Publishing, Cham, Switzerland, 2021), pp. 261–271.
- [46] G. Cencetti, F. Battiston, B. Lepri, and M. Karsai, Temporal properties of higher-order interactions in social networks, *Sci. Rep.* **11**, 7028 (2021).
- [47] J. Noonan and R. Lambiotte, Dynamics of majority rule on hypergraphs, *Phys. Rev. E* **104**, 024316 (2021).
- [48] J. Kim, D.-S. Lee, B. Min, M. A. Porter, M. San Miguel, and K.-I. Goh, Competition between group interactions and non-linearity in voter dynamics on hypergraphs, *Phys. Rev. E* **111**, L052301 (2025).
- [49] L. Hébert-Dufresne, T. M. Waring, G. St-Onge, M. T. Niles, L. K. Corlew, M. P. Dube, S. J. Miller, N. J. Gotelli, and B. J. McGill, Source-sink behavioural dynamics limit institutional evolution in a group-structured society, *R. Soc. Open Sci.* **9**, 211743 (2022).
- [50] G. St-Onge, L. Hébert-Dufresne, and A. Allard, Nonlinear bias toward complex contagion in uncertain transmission settings, *Proc. Nat. Acad. Sci. USA* **121**, e2312202121 (2024).
- [51] J. St-Onge, G. Burgio, S. F. Rosenblatt, T. M. Waring, and L. Hébert-Dufresne, Paradoxes in the coevolution of contagions and institutions, *Proc. R. Soc. B* **291**, 20241117 (2024).
- [52] A. P. Millán, H. Sun, L. Giambagli, R. Muolo, T. Carletti, J. J. Torres, F. Radicchi, J. Kurths, and G. Bianconi, Topology shapes dynamics of higher-order networks, *Nat. Phys.* **21**, 353 (2025).
- [53] P. Cisneros-Velarde and F. Bullo, Multi-group SIS epidemics with simplicial and higher-order interactions, *IEEE Trans. Control Network Syst.* **9**, 695 (2022).
- [54] N. W. Landry and J. G. Restrepo, The effect of heterogeneity on hypergraph contagion models, *Chaos* **30**, 103117 (2020).
- [55] P. S. Skardal and A. Arenas, Higher order interactions in complex networks of phase oscillators promote abrupt synchronization switching, *Commun. Phys.* **3**, 218 (2020).
- [56] S. Adhikari, J. G. Restrepo, and P. S. Skardal, Synchronization of phase oscillators on complex hypergraphs, *Chaos* **33**, 033116 (2023).
- [57] D. A. Prentice and D. T. Miller, Pluralistic ignorance and the perpetuation of social norms by unwitting actors, *Adv. Exp. Soc. Psych.* **28**, 161 (1996).
- [58] D. T. Miller and C. McFarland, Pluralistic ignorance: When similarity is interpreted as dissimilarity, *J. Pers. Social Psych.* **53**, 298 (1987).
- [59] I. L. Janis, Groupthink, *Psych. Today* **5**(November), 43 (1971).
- [60] Y. H. Kureh and M. A. Porter, Fitting in and breaking up: A nonlinear version of coevolving voter models, *Phys. Rev. E* **101**, 062303 (2020).
- [61] C. Bicchieri and H. Mercier, Norms and beliefs: How change occurs, in *The Complexity of Social Norms* (Springer-Verlag, Heidelberg, Germany, 2014), pp. 37–54.
- [62] C. R. Sampson, Oscillatory and excitable opinion dynamics, GitHub (2025), https://github.com/CorbitSampson/Oscillatory_and_Excitable_Opinion_Dynamics.
- [63] K. Nikolay and M. Svetoslav, *Sigmoid Functions: Some Approximation and Modelling Aspects* (Lambert Academic Publishing, London, UK, 2015).
- [64] A. Bizyaeva, A. Franci, and N. E. Leonard, Nonlinear opinion dynamics with tunable sensitivity, *IEEE Trans. Automat. Contr.* **68**, 1415 (2023).
- [65] M. E. J. Newman, *Networks*, 2nd ed. (Oxford University Press, Oxford, UK, 2018).
- [66] I. Z. Kiss, J. C. Miller, and P. L. Simon, *Mathematics of Epidemics on Networks: From Exact to Approximate Models* (Springer International Publishing, Cham, Switzerland, 2017).
- [67] G. F. de Arruda, A. Aleta, and Y. Moreno, Contagion dynamics on higher-order networks, *Nat. Rev. Phys.* **6**, 468 (2024).
- [68] P. S. Chodrow, Configuration models of random hypergraphs, *J. Complex Networks* **8**, cnaa018 (2020).
- [69] S. Melnik, M. A. Porter, P. J. Mucha, and J. P. Gleeson, Dynamics on modular networks with heterogeneous correlations, *Chaos* **24**, 023106 (2014).
- [70] R. Barrio, S. Coombes, M. Desroches, F. Fenton, S. Luther, and E. Pueyo, Excitable dynamics in neural and cardiac systems, *Commun. Nonlinear Sci. Numer. Simul.* **86**, 105275 (2020).
- [71] J. D'Errico, Adaptive Robust Numerical Differentiation (2014), version 1.6, available at <https://www.mathworks.com/matlabcentral/fileexchange/13490-adaptive-robust-numerical-differentiation>.
- [72] M. A. Porter and J. P. Gleeson, Dynamical systems on networks: A tutorial, in *Frontiers in Applied Dynamical Systems: Reviews and Tutorials* (Springer International Publishing, Cham, Switzerland, 2016), Vol. 4.

- [73] N. W. Landry and J. G. Restrepo, Hypergraph assortativity: A dynamical systems perspective, *Chaos* **32**, 053113 (2022).
- [74] N. W. Landry and J. G. Restrepo, Opinion disparity in hypergraphs with community structure, *Phys. Rev. E* **108**, 034311 (2023).
- [75] M. Mäs, Challenges to simulation validation in the social sciences. A critical rationalist perspective, in *Computer Simulation Validation: Fundamental Concepts, Methodological Frameworks, and Philosophical Perspectives*, edited by C. Beisbart and N. J. Saam (Springer International Publishing, Cham, Switzerland, 2019), pp. 857–879.
- [76] J. B. Bak-Coleman, M. Alfano, W. Barfuss, C. T. Bergstrom, M. A. Centeno, I. D. Couzin, J. F. Donges, M. Galesic, A. S. Gersick, J. Jacquet, A. B. Kao, R. E. Moran, P. Romanczuk, D. I. Rubenstein, K. J. Tombak, J. J. Van Bavel, and E. U. Weber, Stewardship of global collective behavior, *Proc. Natl. Acad. Sci. USA* **118**, e2025764118 (2021).
- [77] I. Iacopini, M. Karsai, and A. Barrat, The temporal dynamics of group interactions in higher-order social networks, *Nat. Commun.* **15**, 7391 (2024).
- [78] L. Torres, A. S. Blevins, D. Bassett, and T. Eliassi-Rad, The why, how, and when of representations for complex systems, *SIAM Rev.* **63**, 435 (2021).
- [79] B. K. Fosdick, D. B. Larremore, J. Nishimura, and J. Ugander, Configuring random graph models with fixed degree sequences, *SIAM Rev.* **60**, 315 (2018).
- [80] M. E. J. Newman, Random graphs with clustering, *Phys. Rev. Lett.* **103**, 058701 (2009).
- [81] J. C. Miller, Percolation and epidemics in random clustered networks, *Phys. Rev. E* **80**, 020901(R) (2009).