Homework 3: Due Tuesday April 22, 1pm

1. (20 pts). (Review Problem). Recall that a *partial order* $\leq_P$ on a set S is a relation such that for every $x, y, z \in S$, we have:

   - $x \leq_P x$
   - if $x \leq_P y$ and $y \leq_P x$, then $x = y$,
   - if $x \leq_P y$ and $y \leq_P z$, then $x \leq_P z$.

   Recall that a partial order $\leq_P$ on a set S is said to be *linear* if for every $x, y \in S$, we have either $x \leq_P y$ or $y \leq_P x$.

   (a) Show that for every partial order $\leq_P$ on a finite set S, there is a linear order $\leq_L$ on S such that $\leq_L$ extends $\leq_P$. That is, for every $x, y \in S$, if $x \leq_P y$, then $x \leq_L y$.

   (b) Use König's lemma and part (i) to show that for every partial order $\leq_P$ on an infinite set $S = \{x_0, x_1, \ldots\}$, there is a linear order $\leq_L$ on S extending $\leq_P$.

2. (10 pts) Prove the correctness of the algorithm given in class for efficiently converting a formula ϕ into a formula ψ in CNF which is satisfiable iff ϕ is.

3. (8 pts) Using resolution, show that the set of clauses

$$\{\{p, q, r\}, \{\neg p, \neg q, \neg r\}, \{\neg p, q, s\}, \{p, \neg q, \neg s\},$$
$$\{p, \neg r, s\}, \{\neg p, r, \neg s\}, \{\neg q, r, s\}, \{q, \neg r, \neg s\}\}$$

   is unsatisfiable.

4. (15 pts) (No collaboration) Using our Hilbert-style formal proof system, show that:

   (a) From $\{\neg p \to q, \neg p \to \neg q, p \to (r \to s), p \to r\}$ there is a formal proof of s.

   (b) For every formula ϕ, there is a formal proof of $\neg\neg\phi \to \phi$.