Homework 2: Due Tuesday April 15, 1pm

1. (10 pts). (Review Problem). Say that a formula ϕ is in *conjunctive normal form* (CNF) if ϕ is of the form $\phi = \psi_1 \wedge \psi_2 \wedge \ldots \wedge \psi_n$, where each ψ_i is of the form $\psi_i = \ell_{i,1} \vee \ldots \vee \ell_{i,k_i}$, where each $\ell_{i,j}$ is a *literal*, i.e. either a propositional variable p_m or its negation $\neg p_m$. For example, the formula $(p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is in CNF.

   Show that every formula is equivalent to a formula in conjunctive normal form.

2. (15 pts) Suppose T is a finitely splitting tree having a single infinite path. Suppose also that there is an algorithm for computing T. That is, for each vertex v of T, the algorithm takes v as input and outputs the set of all children of v.

   Describe an algorithm for computing the single infinite path $v_0, v_1, v_2, \ldots$ in T.

3. (**Not required. You may do this problem for a small amount of extra credit.**) Use König's lemma to prove the following: for every graph on infinitely many vertices $\{v_1, v_2, \ldots\}$, there is an infinite subset $S = \{v_{i_1}, v_{i_2}, \ldots\}$ so that either:

   - There is an edge between every two distinct vertices v_{i_j} and v_{i_k} in S,
   - or there is no edge between every two distinct vertices v_{i_j} and v_{i_k} in S.

4. (15 pts) (No collaboration) Let $S = \{\phi_1, \phi_2, \phi_3, \ldots\}$ be an infinite set of formulas. Assume for every valuation v of the variables in the formulas of S, there is some n (depending on v) such that ϕ_n is true for this valuation v. Prove that there is some fixed m such that $\phi_1 \vee \phi_2 \vee \ldots \vee \phi_m$ is a tautology.