Homework 4, due Wednesday Feb 4 at 1pm

1. (no collab) Prove the following closure property of Π_n classes:

   (a) (5 pts) If $R(\mathbf{x})$ and $S(\mathbf{x})$ are Π_n relations, then so is $R(\mathbf{x}) \wedge S(\mathbf{x})$.

   (b) (5 pts) If $R(\mathbf{x}, y)$ is a Π_n relation, so is $\exists y < z R(\mathbf{x}, y)$.

2. (20 pts) Prove that for every $n > 0$, every Σ_n set is many-one reducible to $0^{(n)}$.

3. (20 pts) Prove that for every $n > 0$, a set $A \subseteq \mathbb{N}$ is Δ^0_{n+1} iff $A \leq_T 0^{(n)}$. [Hint: a set is computable iff it is r.e. and its complement is r.e.]

4. Let W_n be the nth r.e. subset of $\mathbb{N}$.

   (a) (15 pts) Show that $\{(n, m) : W_n \subseteq W_m\}$ is Π_2 complete (i.e. show both that this set is Π_2, and every Π_2 set is many-one reducible to it).

   (b) (15 pts) Show that $\{(n, m) : W_n \subseteq^* W_m\}$ is Σ_3 complete, where $A \subseteq^* B$ if $A \setminus B$ is finite (i.e. all but finitely many elements of A are elements of B).