1. NOTATIONS AND REMINDERS

1.1. Sobolev spaces

The variable $x = (x_1, \ldots, x_n)$ will denote a point in the space $\mathbf{R}^n$. The differential operator $\partial/\partial x_i$ is denoted D_i and if $j = (j_1, \ldots, j_n)$ is a multi-integer, we write

(1.1)
$$D^j = D_1^{j_1} \ldots D_n^{j_n} = \frac{\partial^{|j|}}{\partial x_n^{j_1} \ldots \partial x_n^{j_n}}$$

where $|j| = j_1 + \ldots + j^n$.

If $j = (0, \ldots, 0)$ $D^j = \mathrm{I}$ = the identity.

We term Ω an open subset of $\mathbf{R}^n$, which may satisfy a regularity property of the type

(1.2) The boundary Γ of Ω is an r-times continuously differential manifold of dimension $n-1$ and Ω is locally situated on one side only of Γ.

If (1.2) holds, we say that Ω belongs to the class $\mathscr{C}^r$.

We denote by $L^\alpha(\Omega)$ $(1 \leqslant \alpha < +\infty)$ (or $L^\infty(\Omega)$) the space of real functions of Ω into $\mathbf{R}$ whose power α is summable for the Lebesgue measure $\mathrm{d}x = \mathrm{d}x_1 \ldots \mathrm{d}x_n$ (or essentially bounded over Ω). It is a Banach space with norm

$$\| f \|_{L^\alpha(\Omega)} = \left| \int_\Omega |f(x)|^\alpha \, \mathrm{d}x \right|^{1/\alpha}$$

(or

$$\| f \|_{L^\infty(\Omega)} = \text{Ess. sup.} |f(x)|).$$

For $\alpha = 2$, we denote the Hilbert scalar product of $L^2(\Omega)$ by

$$(f, g) = \int_\Omega f(x) g(x) \, \mathrm{d}x,$$

and

$$|f| = (f, f)^{1/2} = \| f \|_{L^2(\Omega)}.$$

For m an integer and $1 \leqslant \alpha \leqslant +\infty$, we denote by $W^{m,\alpha}(\Omega)$ the Sobolev space [1] [2] (*cf.* also Lions [2]) of $u \in L^\alpha(\Omega)$, all of whose derivatives of order $\leqslant m$ are in $L^\alpha(\Omega)$. It is a Banach space for the norm

(1.3)
$$\| u \|_{W^{m,\alpha}(\Omega)} = \left| \sum_{|j| \leqslant m} \| D^j u \|^\alpha_{L^\alpha(\Omega)} \right|^{1/\alpha}.$$

For $\alpha = 2$, we write $H^m(\Omega) = W^{m,2}(\Omega)$, and the norm (1.3) is the Hilbert norm corresponding to the scalar product

(1.4)
$$((u, v))_{H^m(\Omega)} = \sum_{|j| \leqslant m} (D^j u, D^j v).$$

The closure in $W^{m,\alpha}(\Omega)$ (or $H^m(\Omega)$) of the subspace of functions with compact support in Ω is denoted by $W_0^{m,\alpha}(\Omega)$ (or $H_0^m(\Omega)$ for $\alpha = 2$). This is also the closure of $\mathscr{D}(\Omega)$, the subspace of functions of Ω into $\mathbf{R}$ which are indefinitely differentiable and of compact support in Ω.

If the open space Ω is regular [e.g. (1.2) with $r = m + 2$], we can define a trace operator

(1.5)
$$\gamma = (\gamma_0, \ldots, \gamma_{m-1})$$

which is linear and (in particular) continuous from $W^{m,\alpha}(\Omega)$ into $[L^\alpha(\Gamma)]^m$ (1), and such that if u is m-times continuously differentiable in $\bar{\Omega}$,

$$\gamma_0 u = u|_\Gamma, \quad \gamma_1 u = \left.\frac{\partial u}{\partial v}\right|_\Gamma, \quad \ldots, \quad \gamma_j u = \left.\frac{\partial^j u}{\partial v^j}\right|_\Gamma,$$

(1) $L^\alpha(\Gamma)$ = the space of functions L^α over Γ for the surface measure $\mathrm{d}\Gamma$.