Math 273: **Homework #5, due on Friday, December 8**
(no late homework accepted)

A take-home final will also be assigned very soon.

[**1**] Consider the problem

$$\min\ \sin(x_1 + x_2) + x_3^2 + \frac{1}{3}(x_4 + x_5^4 + \frac{x_6}{2})$$

subject to
$8x_1 - 6x_2 + x_3 + 9x_4 + 4x_5 = 6$
$3x_1 + 2x_2 - x_4 + 6x_5 + 4x_6 = -4.$
Transform the problem into an unconstrained minimization problem.

[**2**] Show that $(0, -1)^T$ is a local minimizer for the problem
Minimize $f(x) = 2x_1^2 + x_2$ subject to
$x_2 \geq x_1^2 - 1$
$x_1 \geq x_2.$

[**3**] The problem of finding the shortest distance from a point x_0 to the hyperplane $\{x : \ Ax = b\}$ where A has full row rank can be formulated as the quadratic program

$$\min \frac{1}{2}(x - x_0)^T(x - x_0), \quad \text{s.t. } Ax = b.$$

Show that the optimal multiplier is $\lambda^* = (AA^T)^{-1}(b - Ax_0)$, and that the solution is $x_* = x_0 + A^T(AA^T)^{-1}(b - Ax_0)$.

Show that in the special case where A is a row vector, the shortest distance from x_0 to the solution set of $Ax = b$ is $\frac{|b - Ax_0|}{\|A\|}$.

[**4**] Repeat problem [3], Hw #4 using Newton's method, and compare the two methods. Give details about your implementation (computation of gradient, of Hessian, of inverse, about α, about your stopping criteria, etc), and include your code.