Math 269C. Vese. HW #4

[1] Let $K$ be a tetrahedron with vertices $a^i$, $i = 1, ..., 4$, and let $a^{ij}$ denote the midpoint on the straight line $a^i a^j$, $i < j$. Show that a function $v \in P_2(K)$ is uniquely determined by the degrees of freedom: $v(a^i), v(a^{ij}), i, j = 1, ..., 4, i < j$. Show that the corresponding finite element space $V_h$ satisfies $V_h \subset C^0(\Omega)$, assuming continuity at all degrees of freedom.

[2] Let $K$ be a triangle with vertices $a^i$, $i = 1, 2, 3$. Suppose that $v \in P_r(K)$ and that $v$ vanishes on the side $a^2a^3$. Prove that $v$ has the form

$$v(x) = \lambda_1(x) w_{r-1}(x), \quad x \in K,$$

where $w_{r-1} \in P_{r-1}(K)$, and $\lambda_1(x)$ is the affine local basis function corresponding to the node $a^1$.

(for simplicity, you can assume that $K$ is the reference triangle with vertices $(0,0)$, $(0,1)$ and $(1,0)$, and that the side $a^2a^3$ is on one of the axes).

[3] Let $K$ be a triangle with vertices $a^i$, $i = 1, 2, 3$, and let $a^{ij}$, $i < j$, denote the midpoints of the sides of $K$. Let $a^{123}$ denote the center of gravity of $K$. Prove that $v \in P_4(K)$ is uniquely determined by the following degrees of freedom

$$v(a^i),$$
$$\frac{\partial v}{\partial x_j}(a^i), \quad i = 1, 2, 3, \quad j = 1, 2,$$
$$v(a^{ij}), \quad i, j = 1, 2, 3, \quad i < j,$$
$$v(a^{123}), \quad \frac{\partial v}{\partial x_j}(a^{123}), \quad j = 1, 2,$$

(typo in Exercise 3.8 in the textbook).

Also show that the functions in the corresponding finite element space $V_h$ are continuous, assuming continuity for all degrees of freedom.